Constructing and Routing on Plane Constant Spanners

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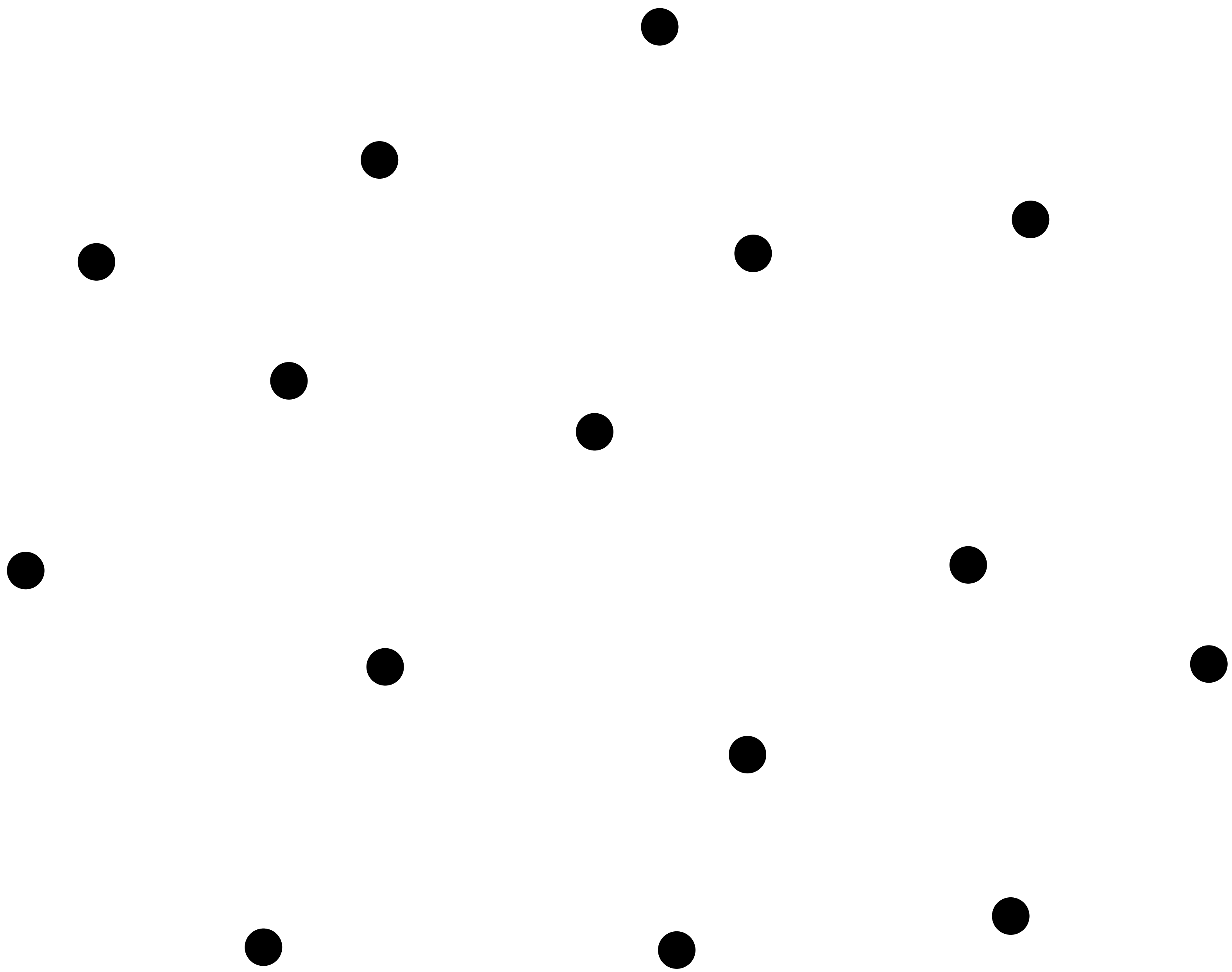
Ottawa, Canada

What is a Geometric Graph?



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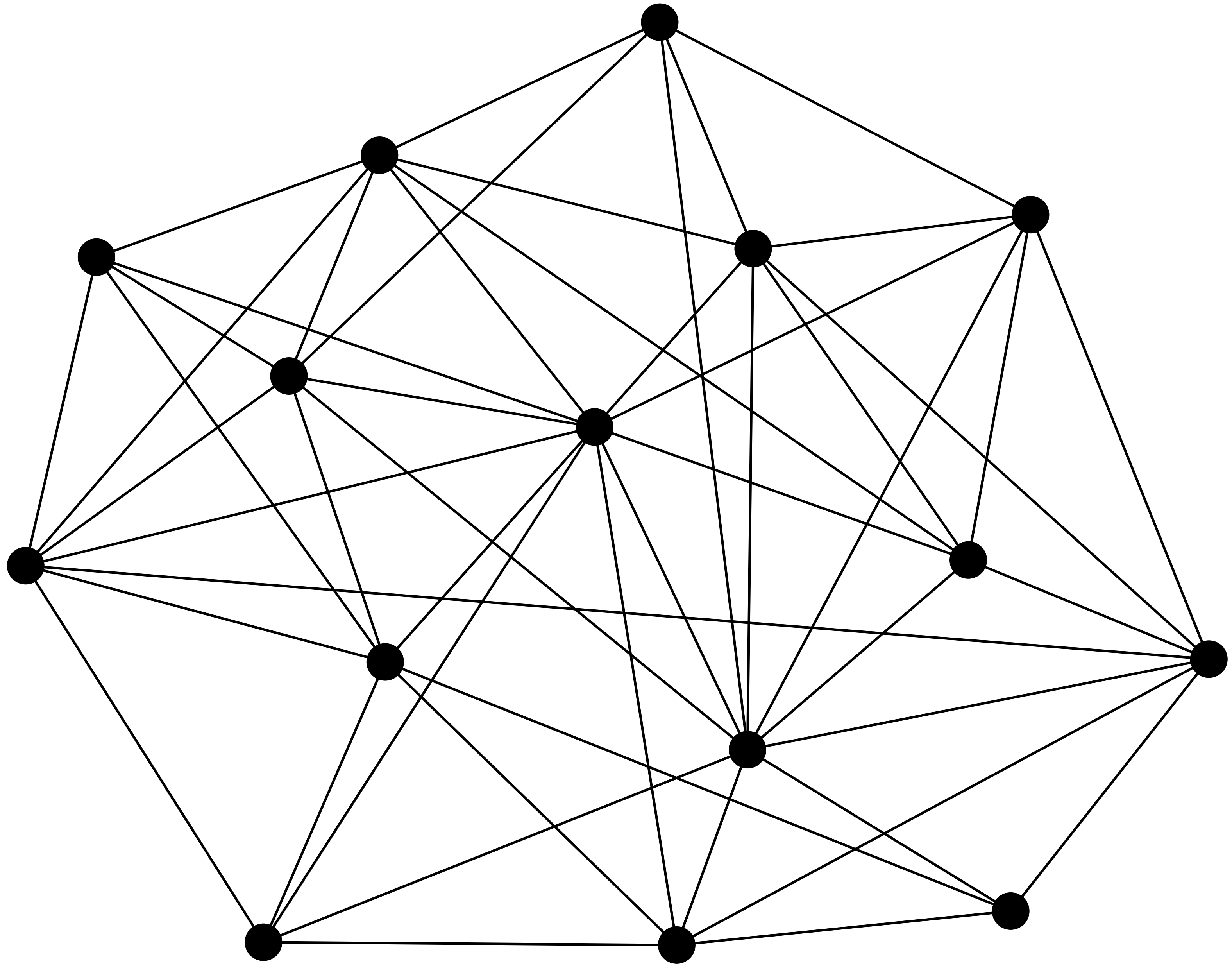

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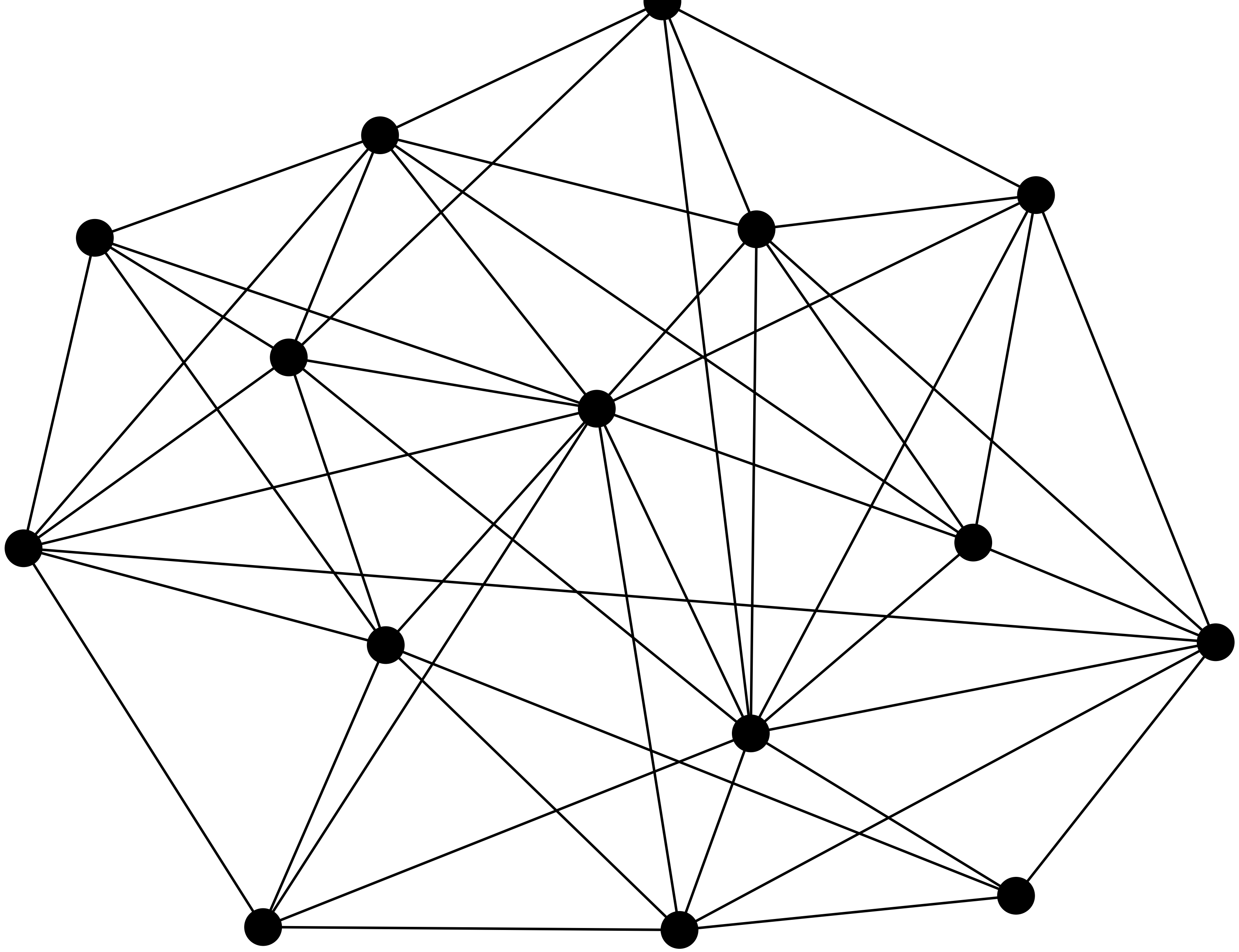

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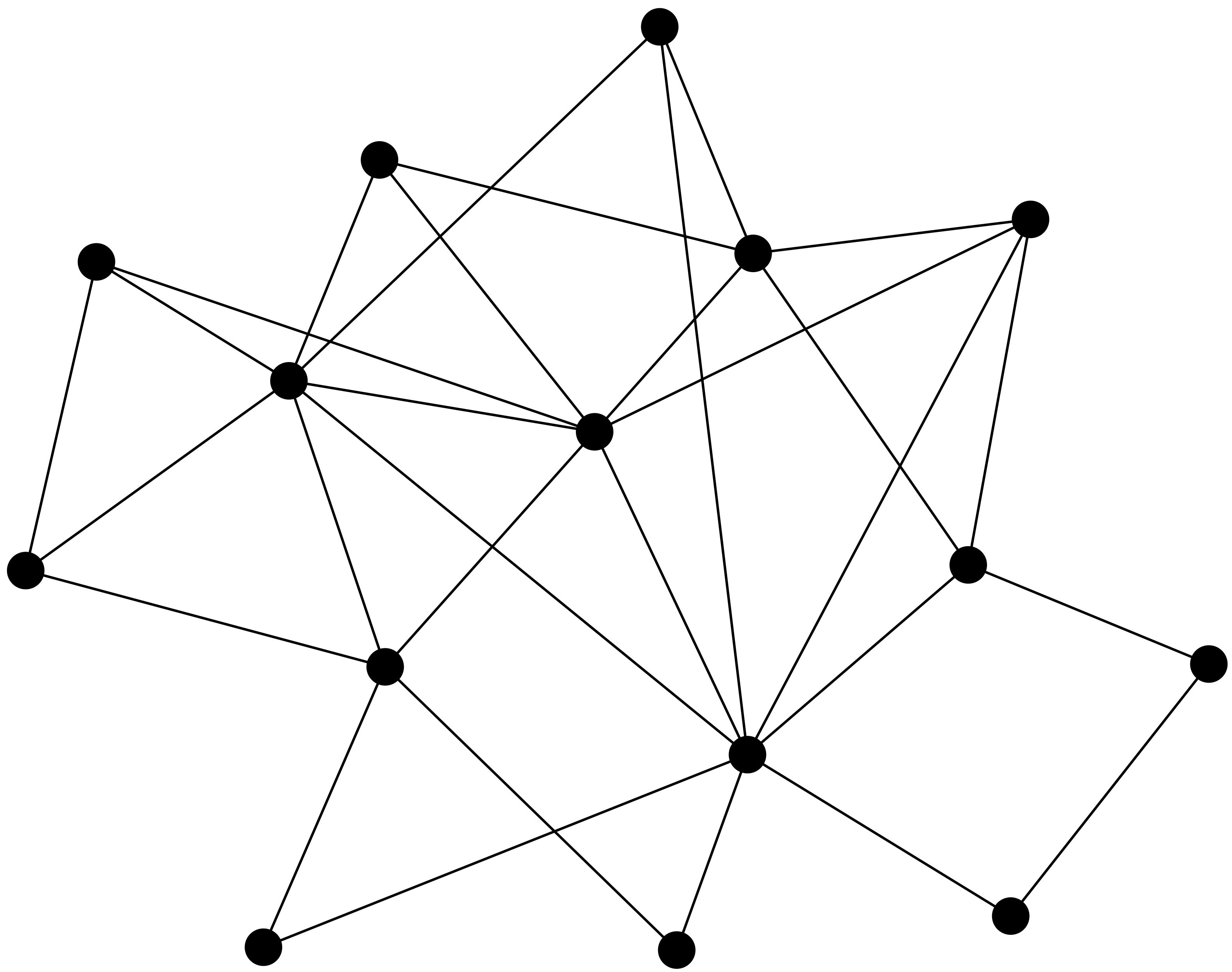

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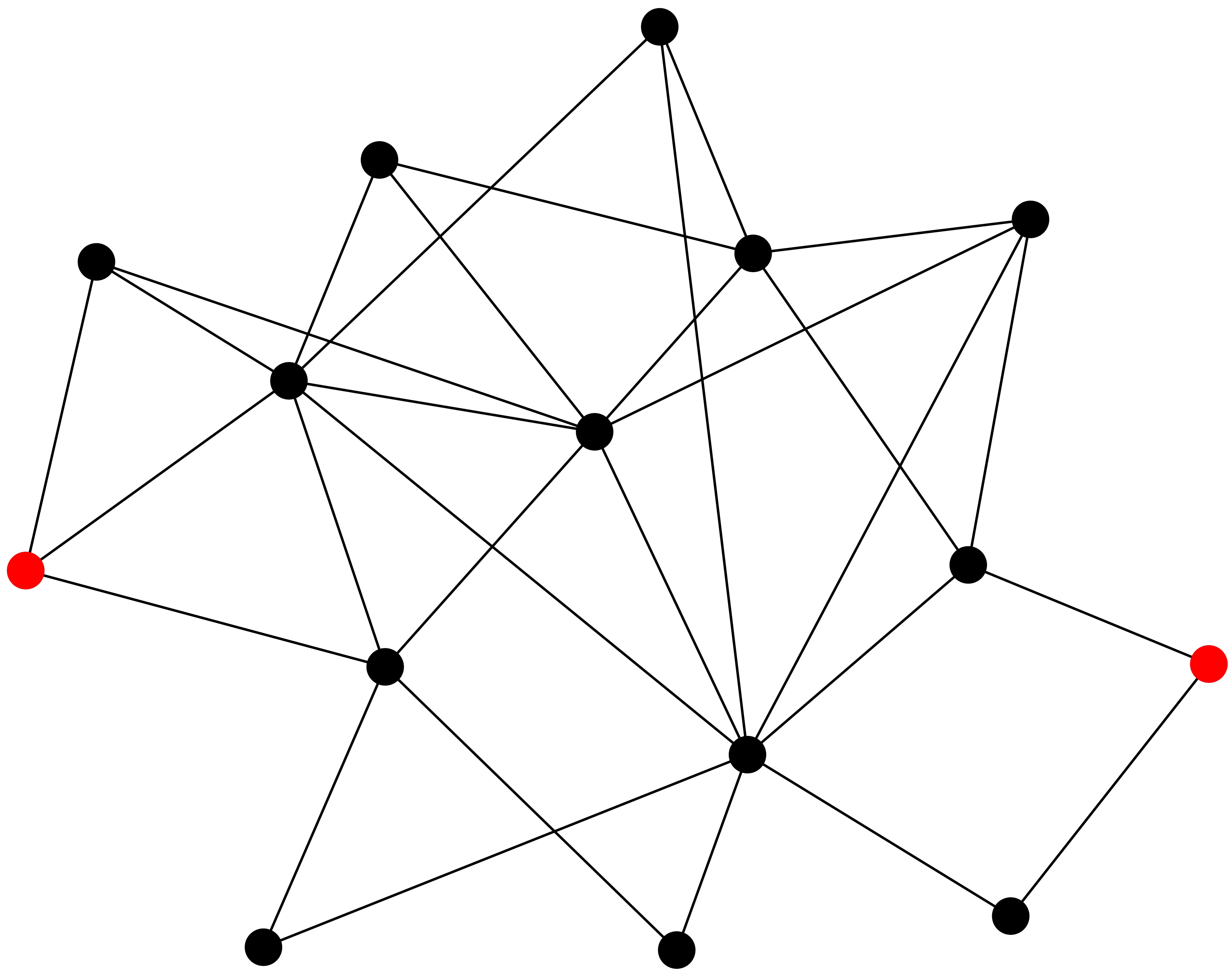

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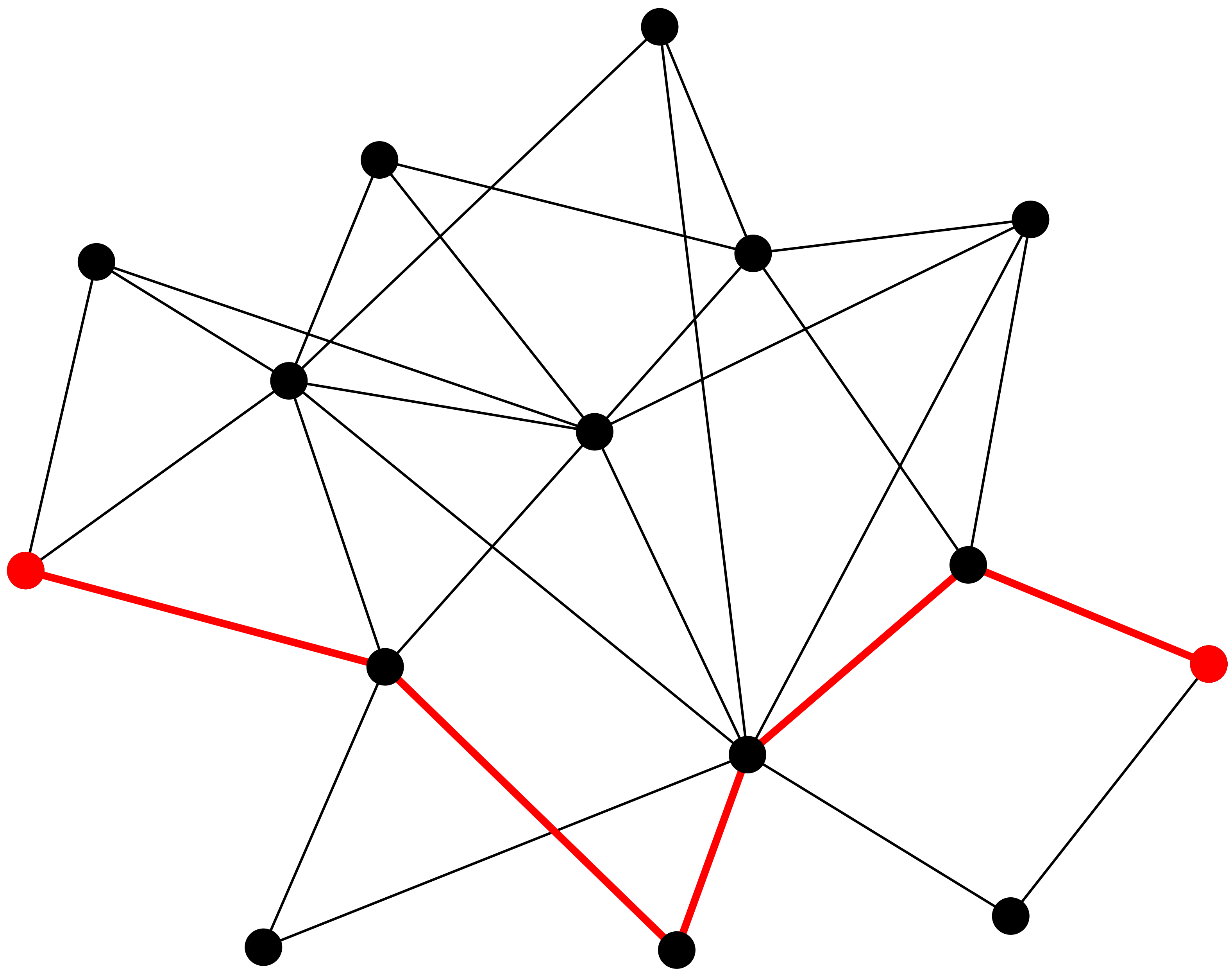

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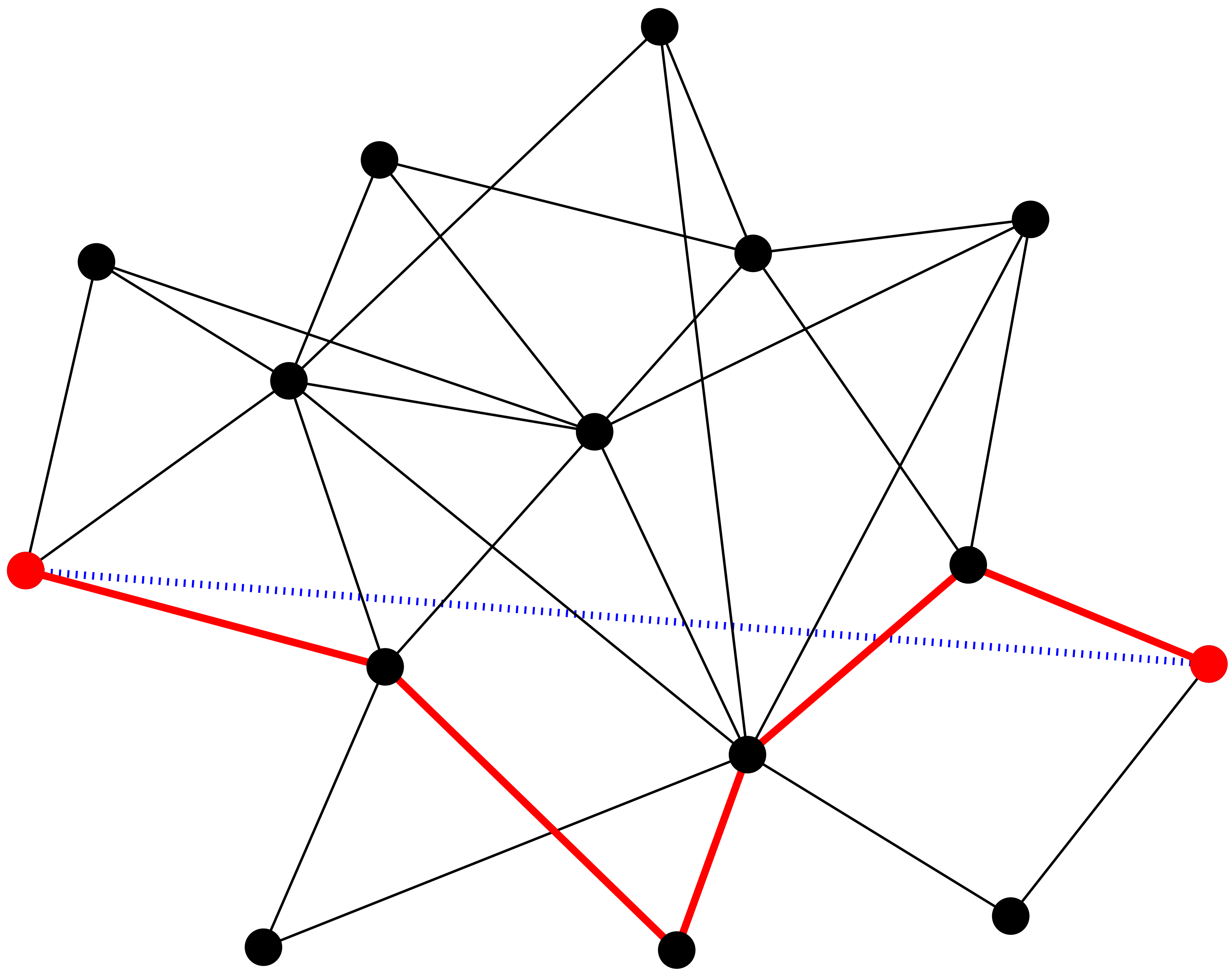


What is a t-spanner?

Given a geometric graph $G = (V, E)$. A spanning subgraph $G' = (V, E')$ is a t-spanner of G for constant $t \geq 1$ provided we have:

$$\forall xy \in E(G),\ d_{G'}(x, y) \leq t \cdot |xy|$$

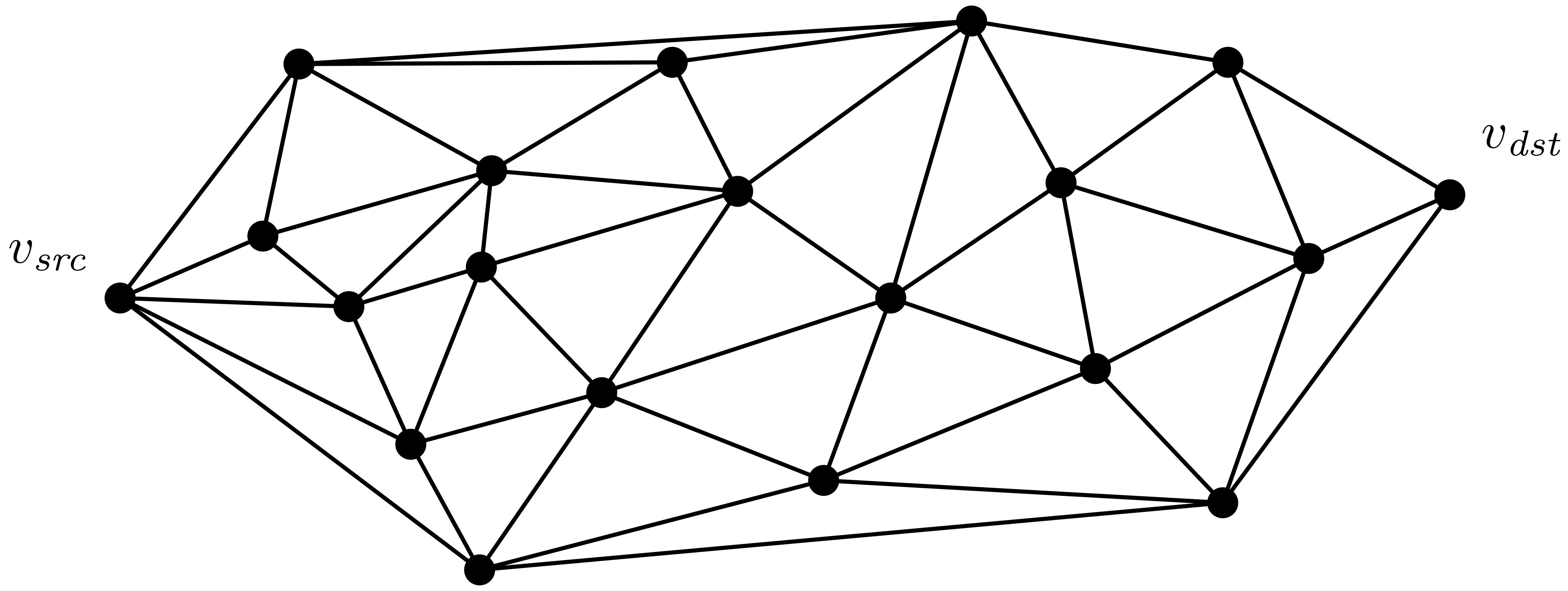
What is a t-spanner?

Sample goals for spanning subgraphs:

1. small spanning ratio
2. sparse graph
3. low maximum degree
4. high connectivity
5. low weight
6. robust (i.e. fault tolerant)
7. small diameter

Online Routing

Given a geometric graph G, find a path from a vertex v_{src} to a vertex v_{dst} *online*.



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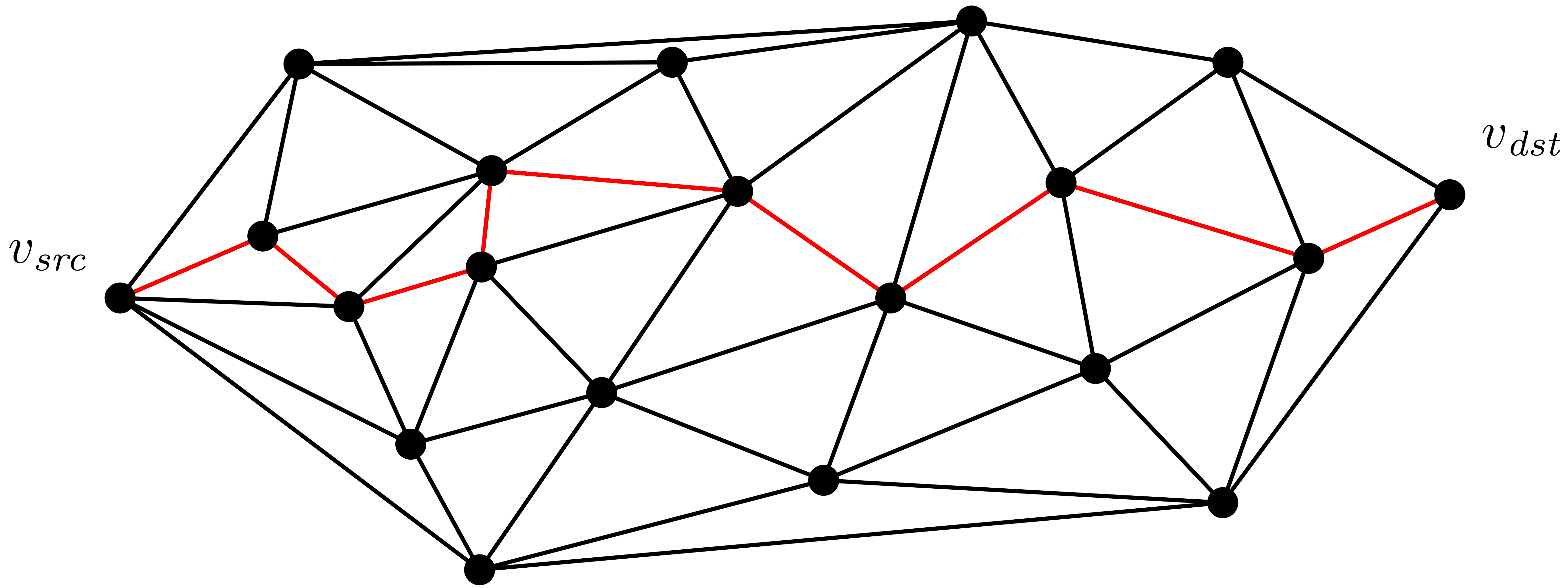

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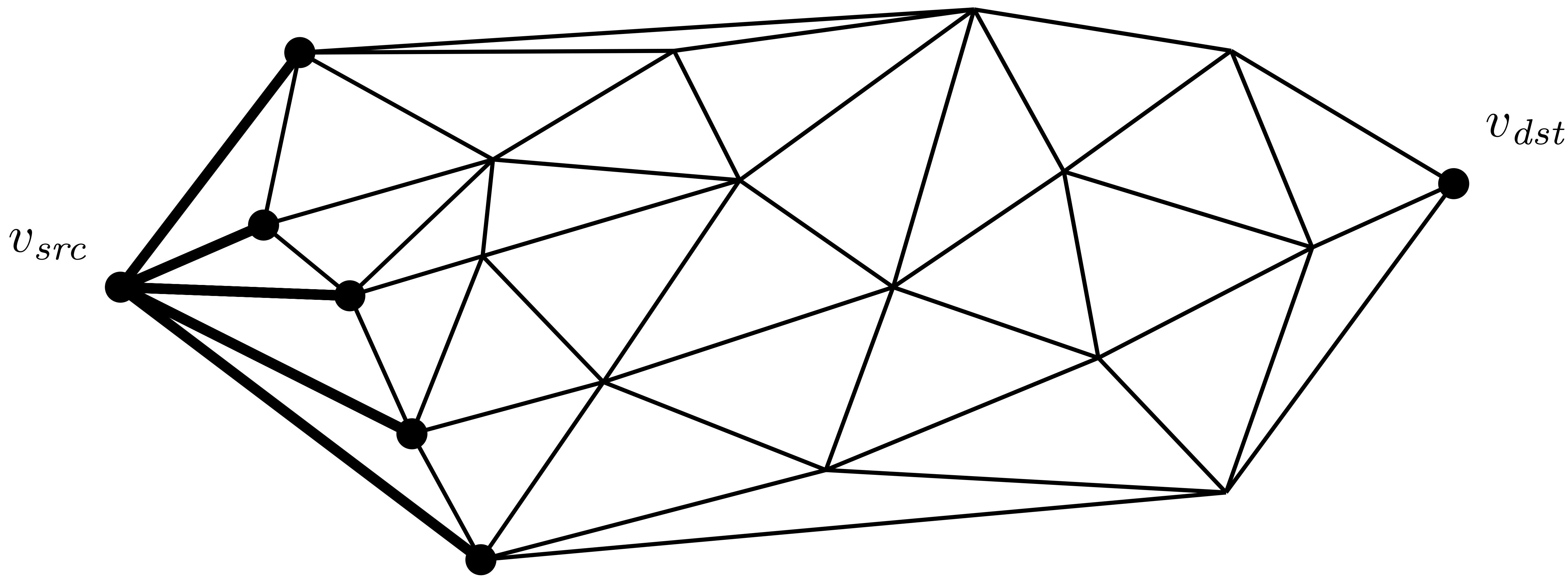

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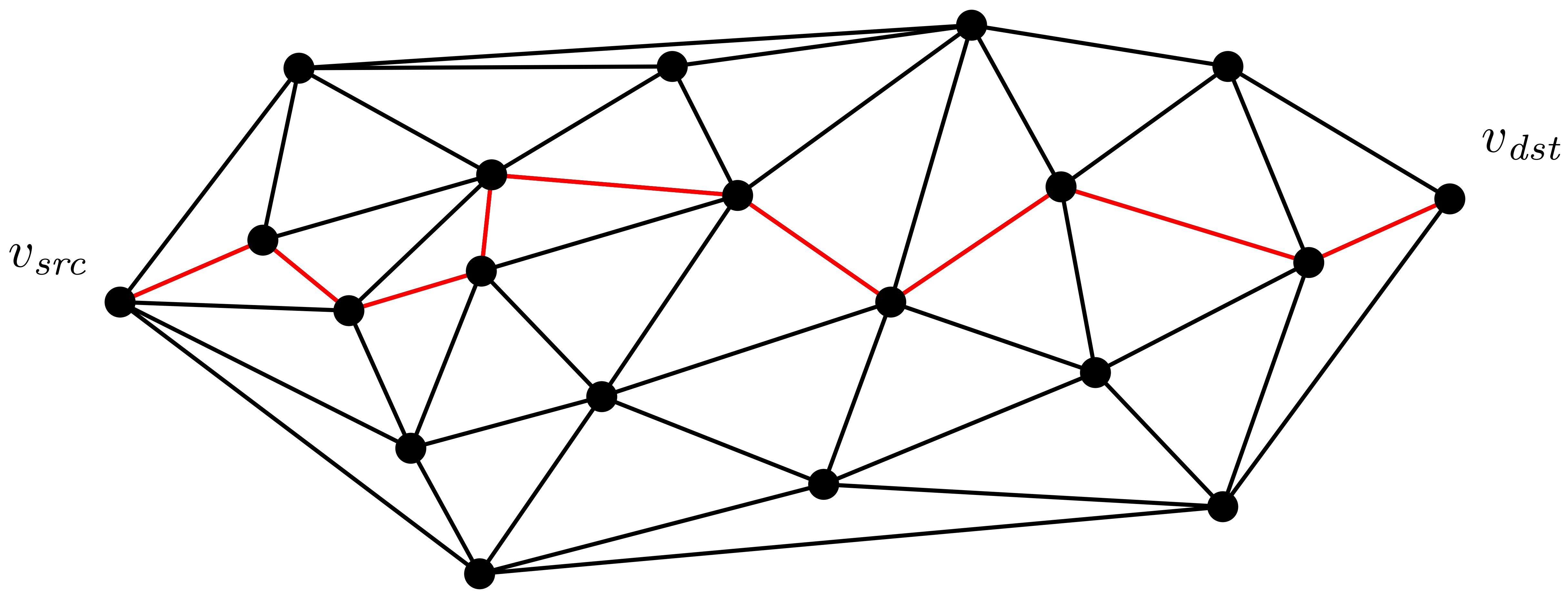

Online Routing

In the weakest model, the routing algorithm must decide given:

1. v_{cur}, the current vertex,
2. v_{dst}, the destination, and
3. $N(v_{cur})$, the neighbors of v_{cur}.

An online strategy is *oblivious* if it makes a decision based only on the above information (memoryless).

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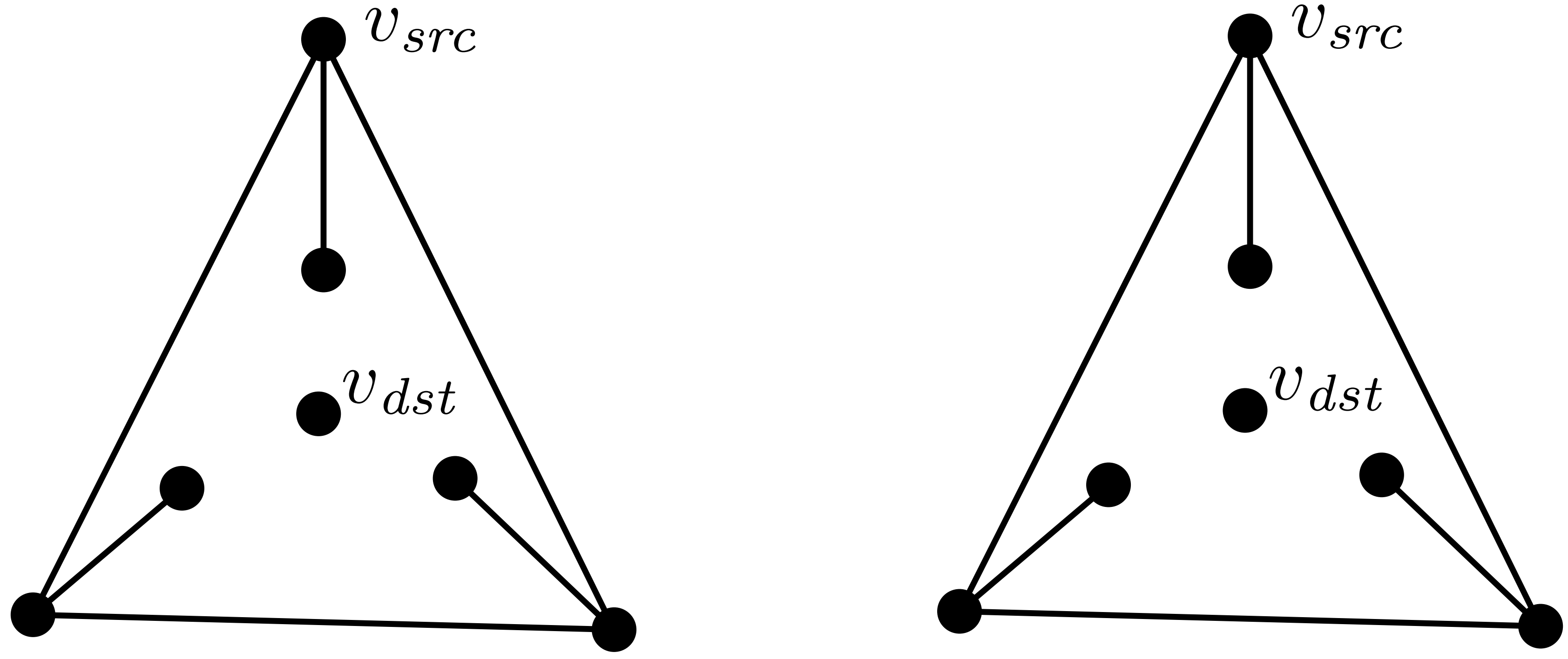

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Models are sometimes strengthened to include:

1. The routing algorithm has some local memory
2. Additional bits of information are stored with v_{cur}

Online Routing

The routing ratio is essentially the worst spanning ratio of the path returned by a routing algorithm.

Goals for routing algorithms:

1. Low routing ratio
2. Use of little or no local memory
3. Require little information stored with the vertices
4. Low number of edges in the path (when one exists)
5. Simple

Online Routing

The routing ratio is essentially the worst spanning ratio of the path returned by a routing algorithm.

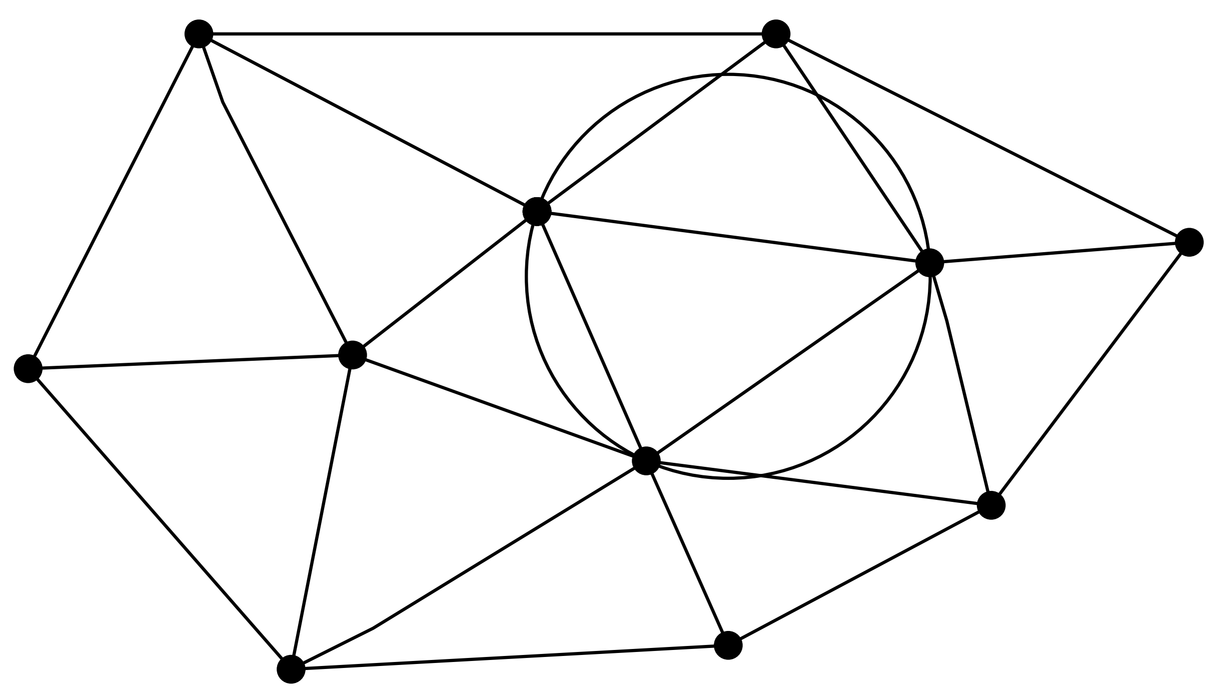
Want to use the geometry of the graph to compete against routing tables. Also, by exploiting the geometry, we make the algorithms easier to adapt in a dynamic setting.

Focus for this talk

1. Construction of plane constant spanners
2. Routing algorithms on plane constant spanners

Question:

Does the standard Delaunay triangulation of a point set have constant spanning ratio?



Standard Delaunay triangulation is a constant spanner:

Dobkin, Friedman and Supowit (1990) showed that the Delaunay triangulation is a 5.08-spanner.

Standard Delaunay triangulation is a constant spanner:

One-sided Voronoi Paths:

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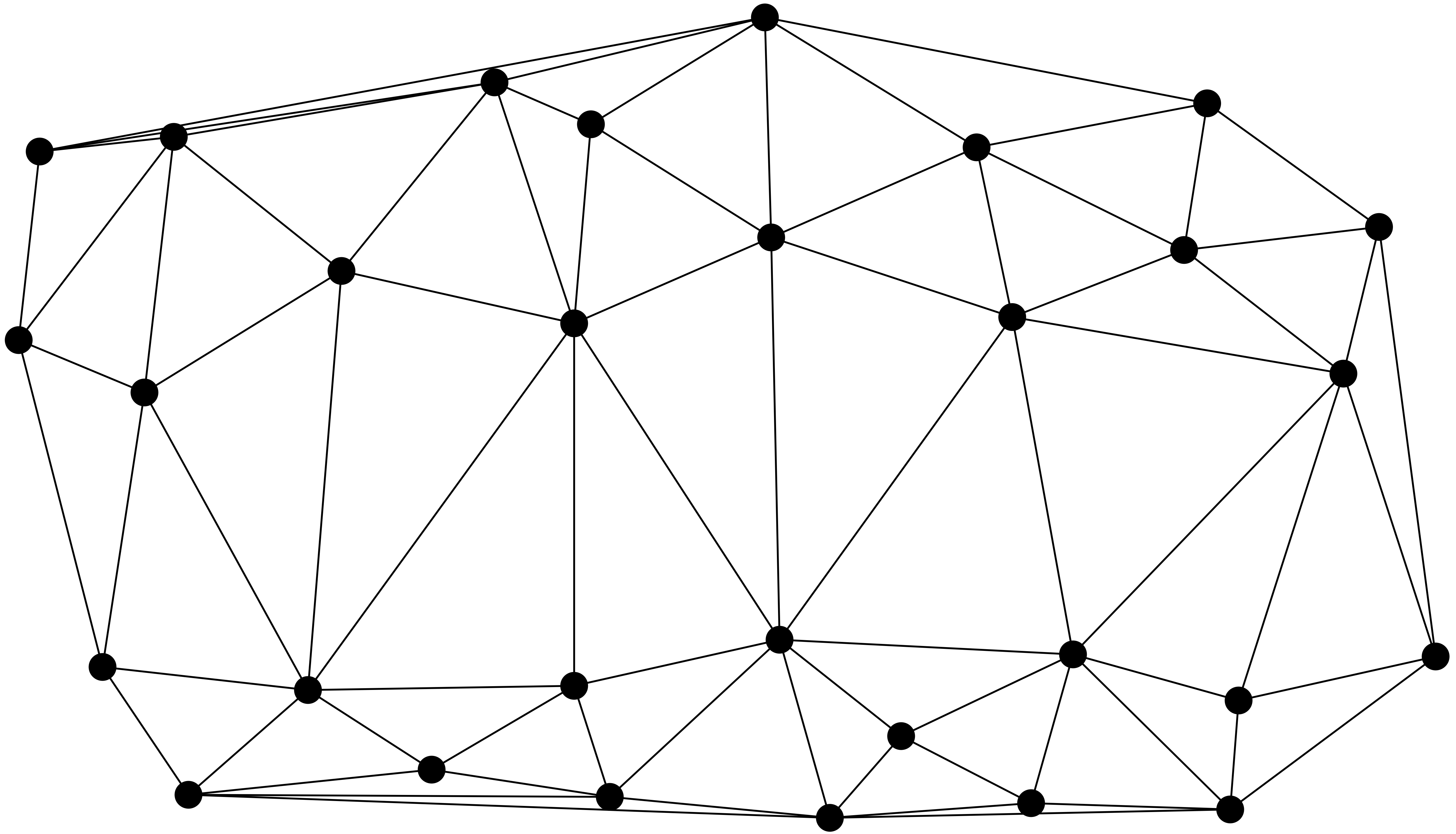

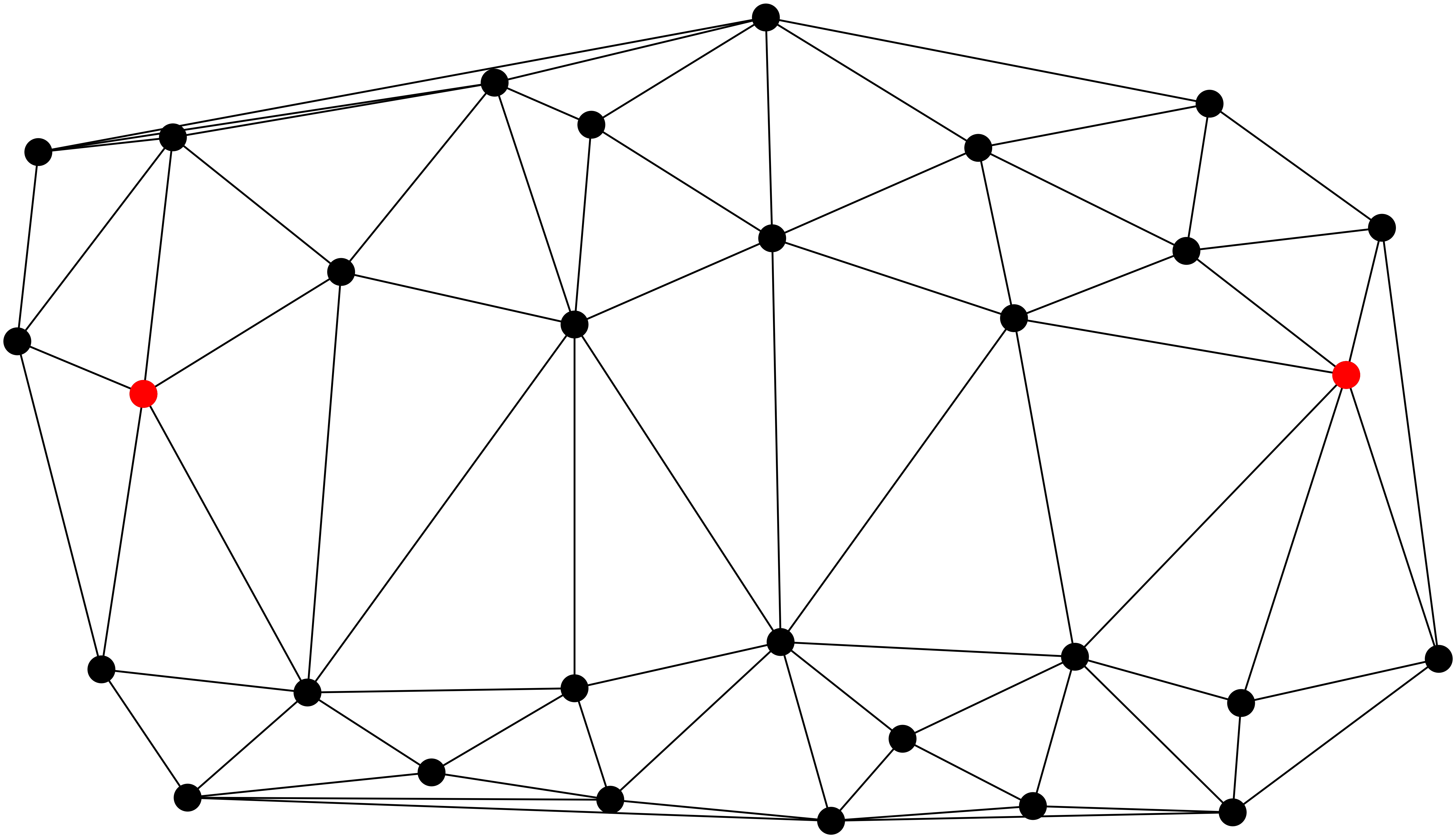

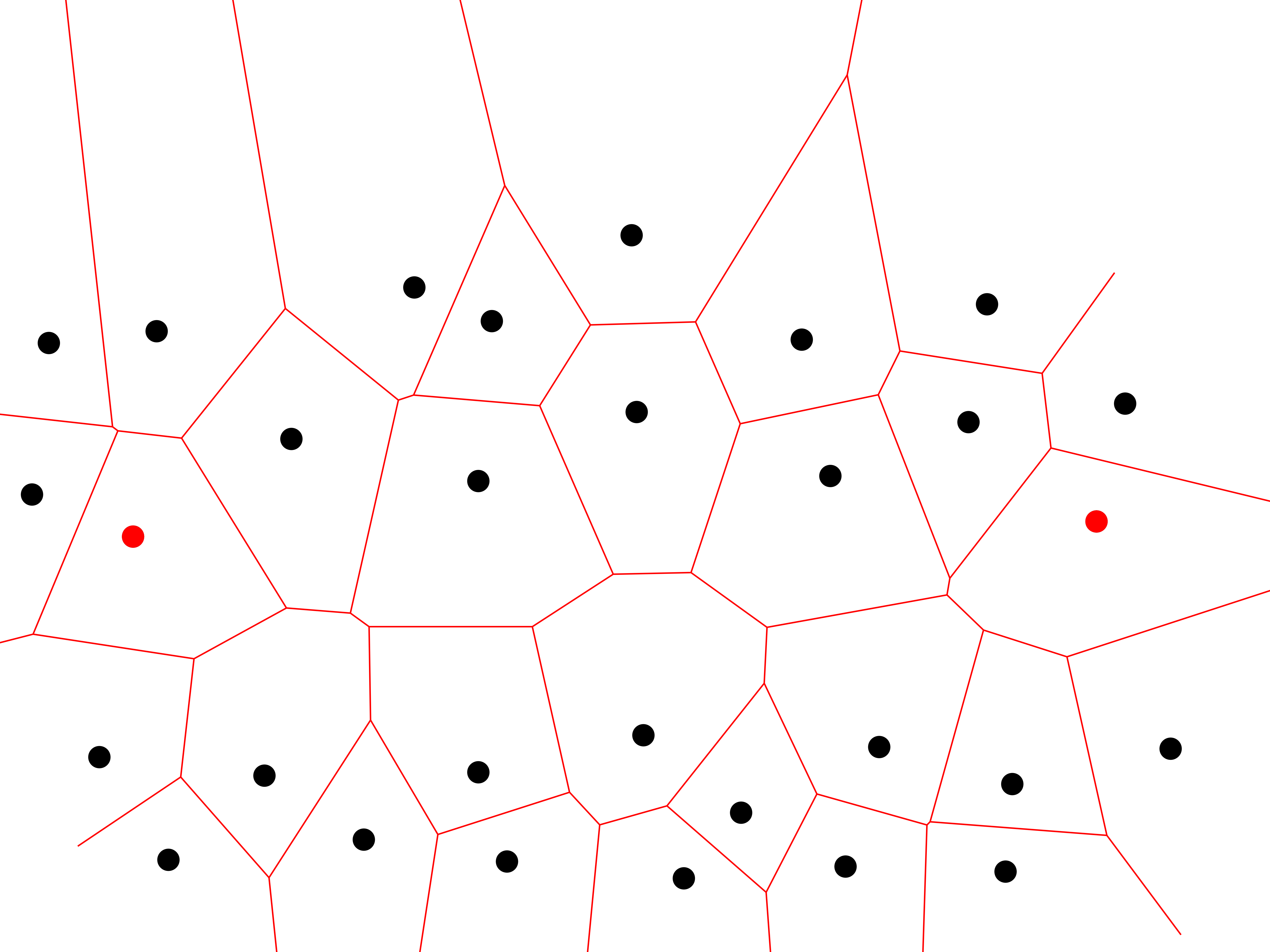

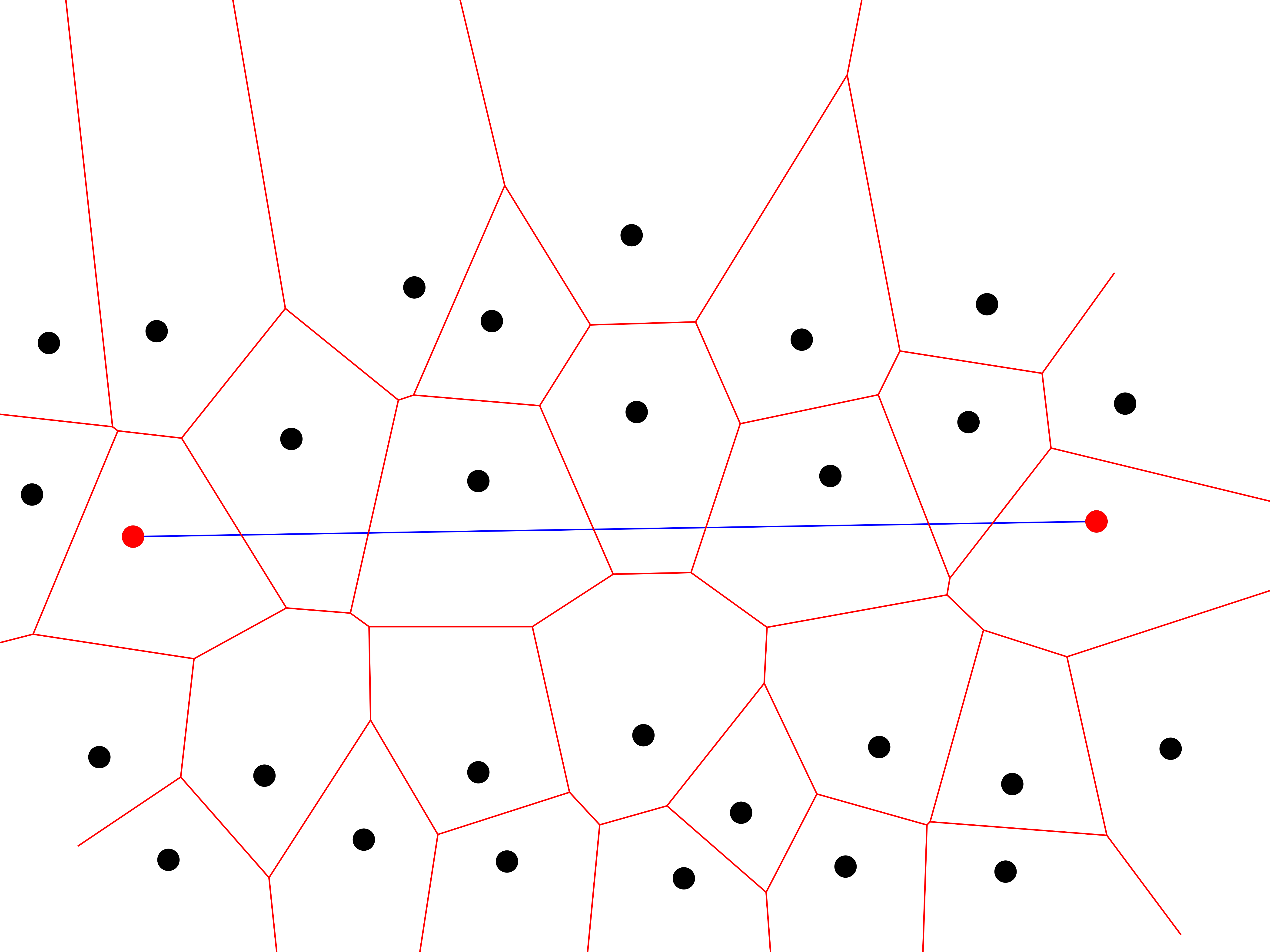

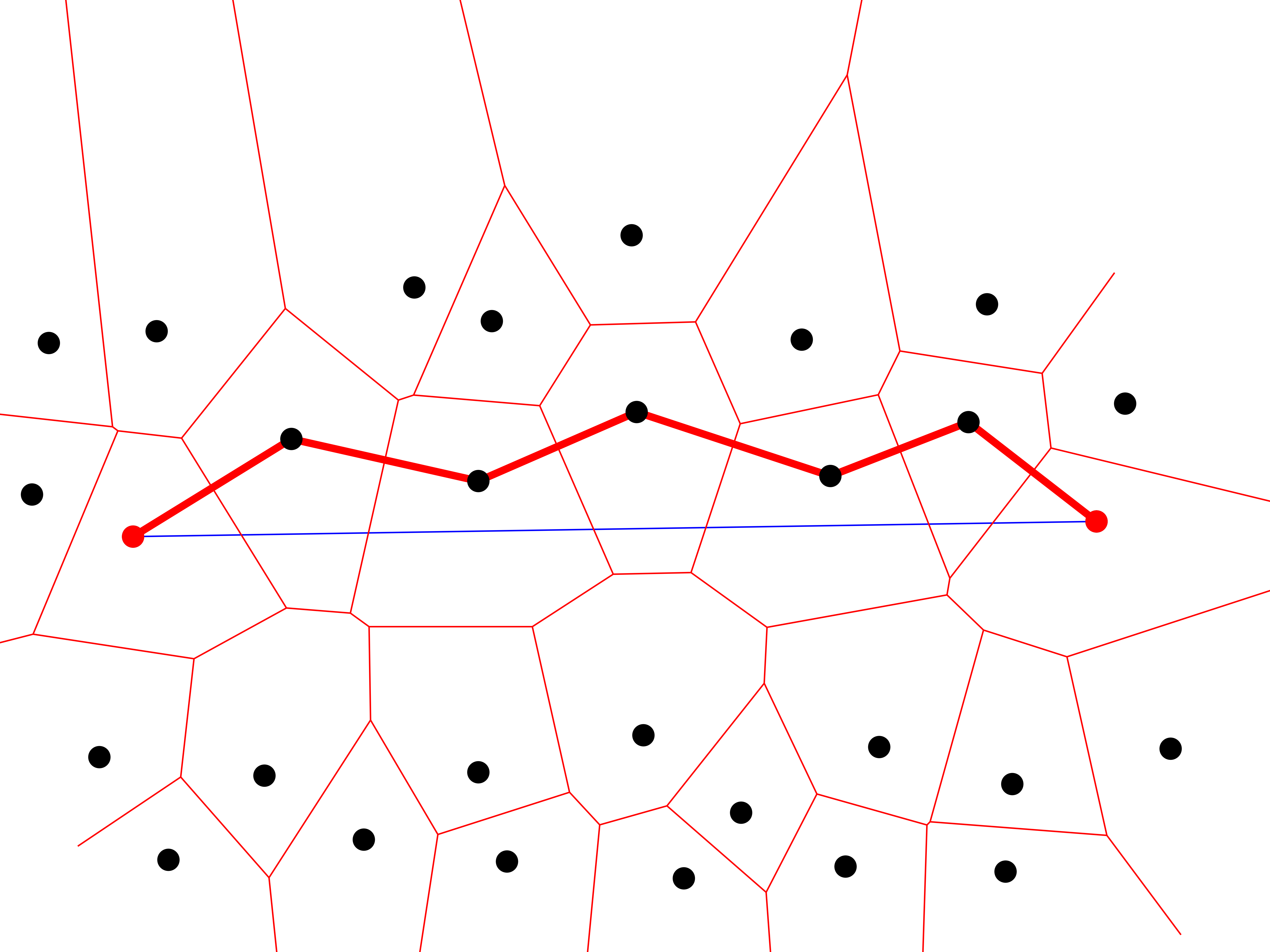

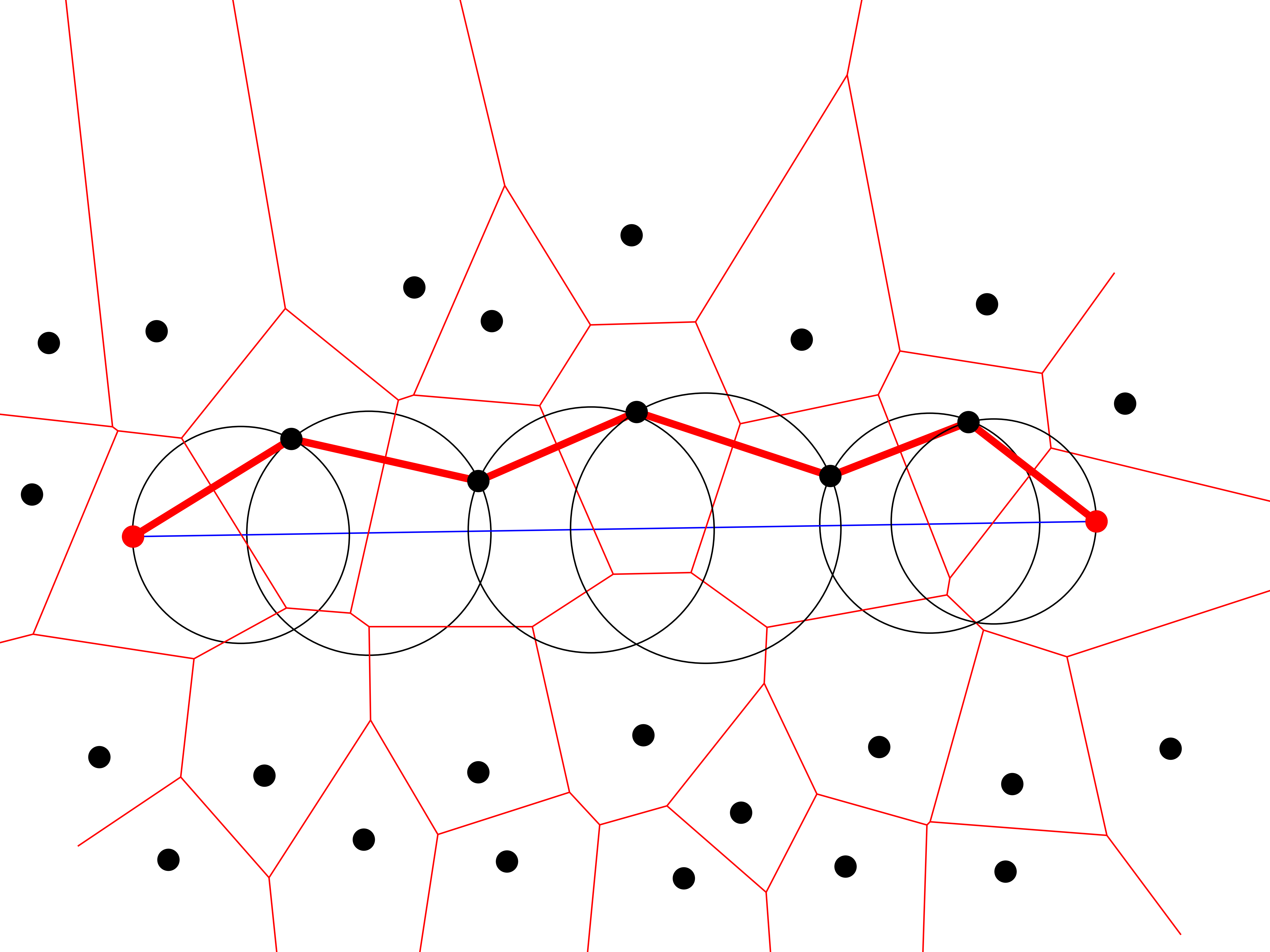

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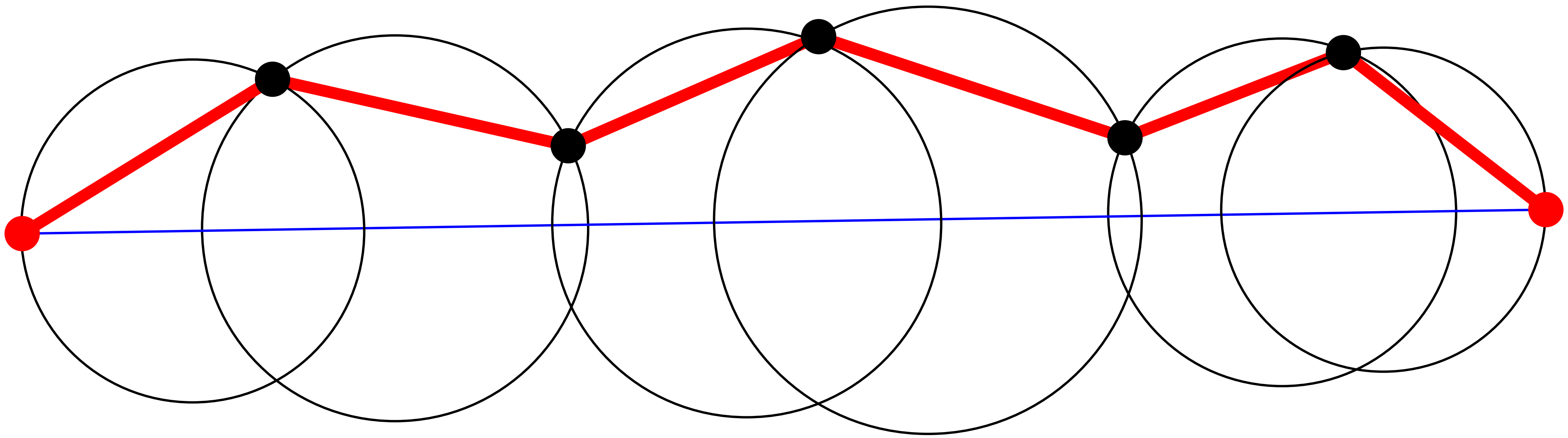

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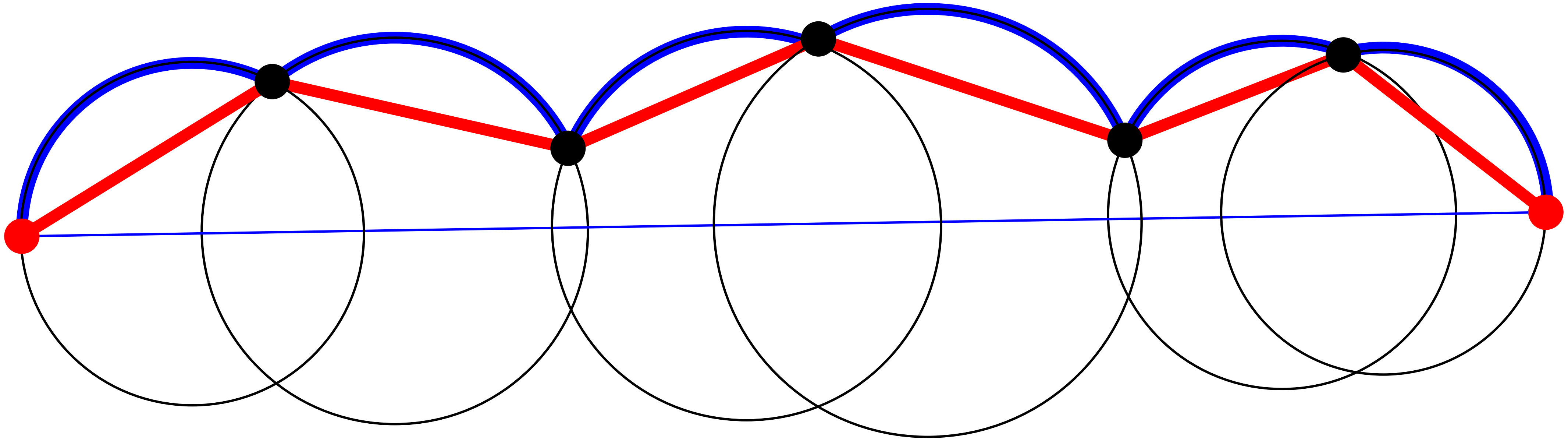

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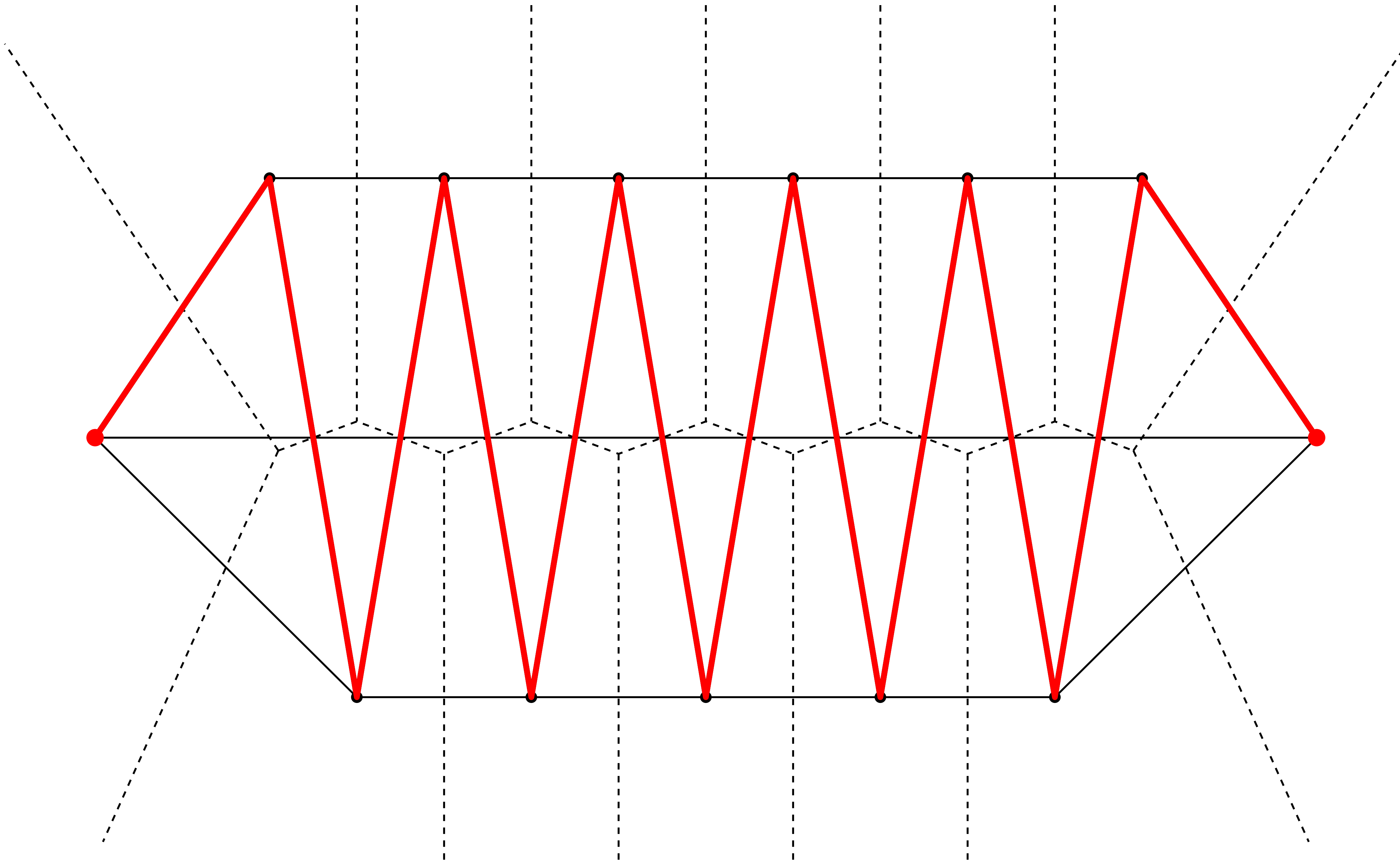

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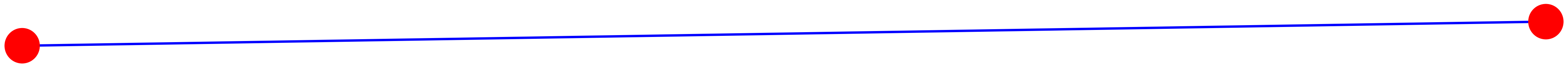

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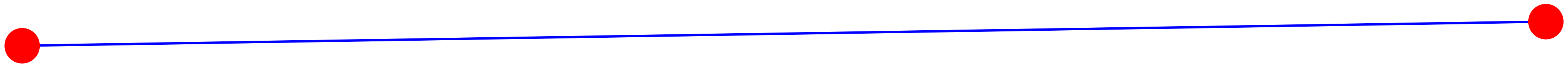

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- When a one-sided Voronoi path does not exist, then shortcuts need to be taken.
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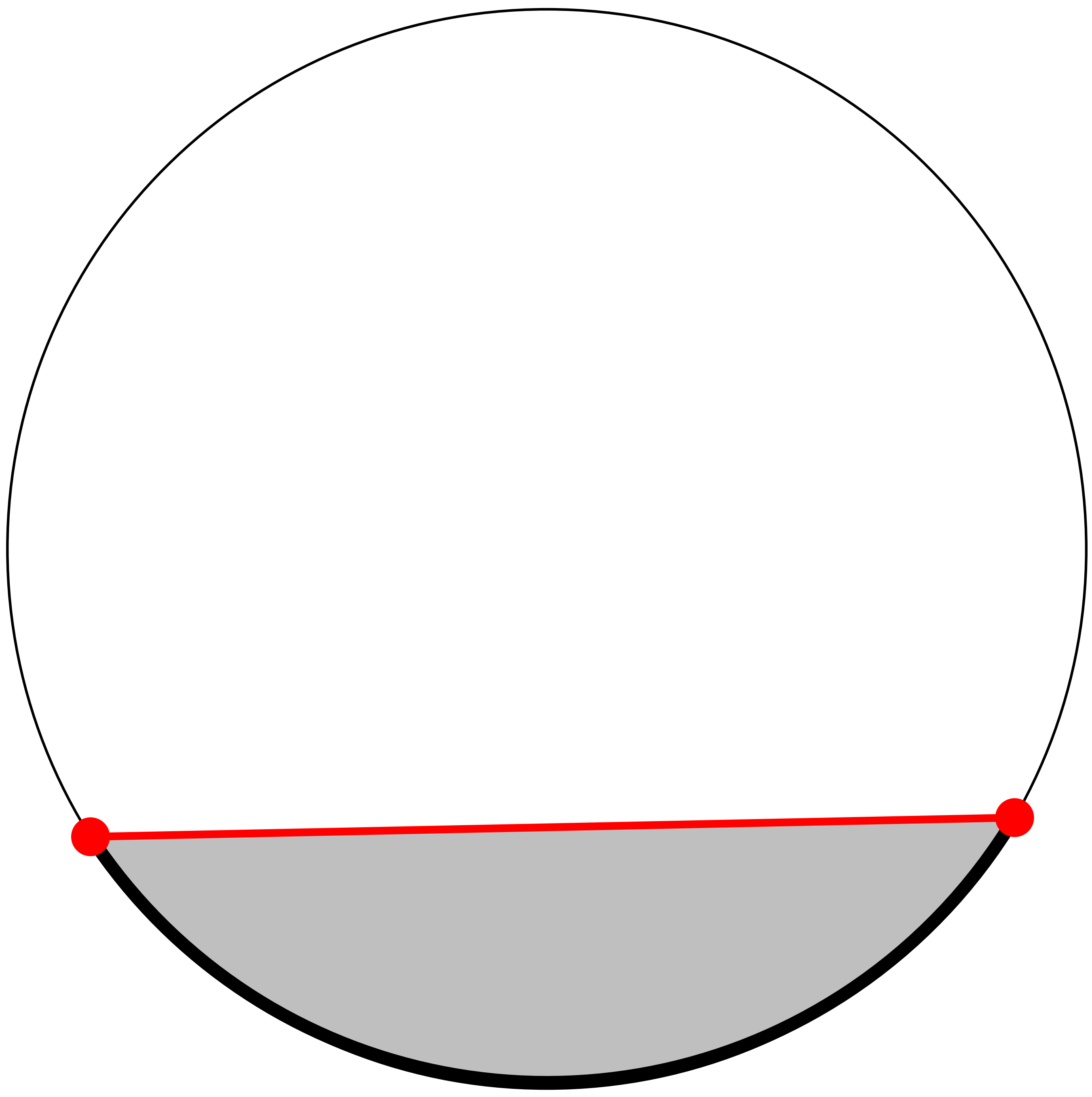
Keil and Gutwin's result:



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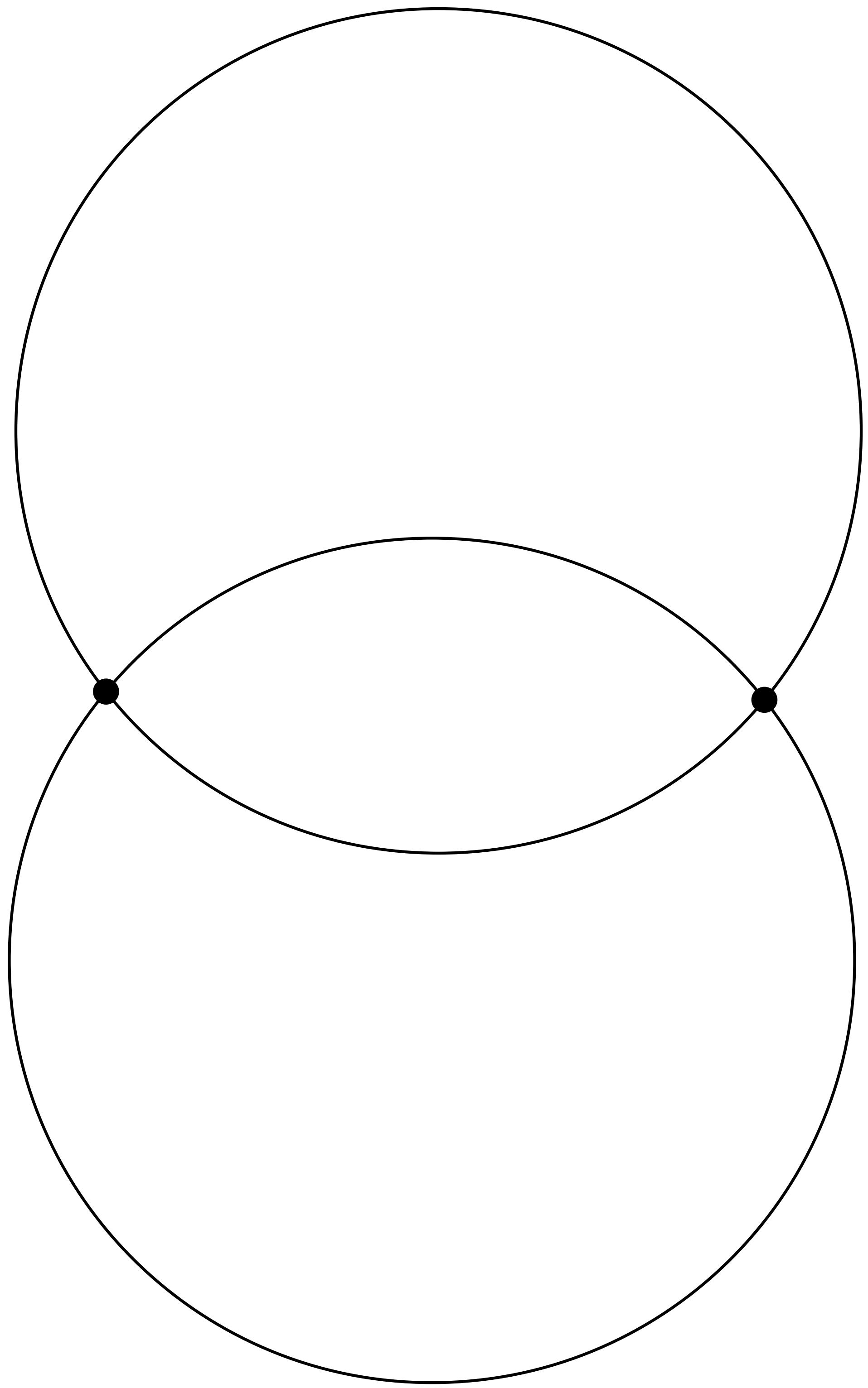

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- Xia (2011) shows a bound of 1.998.

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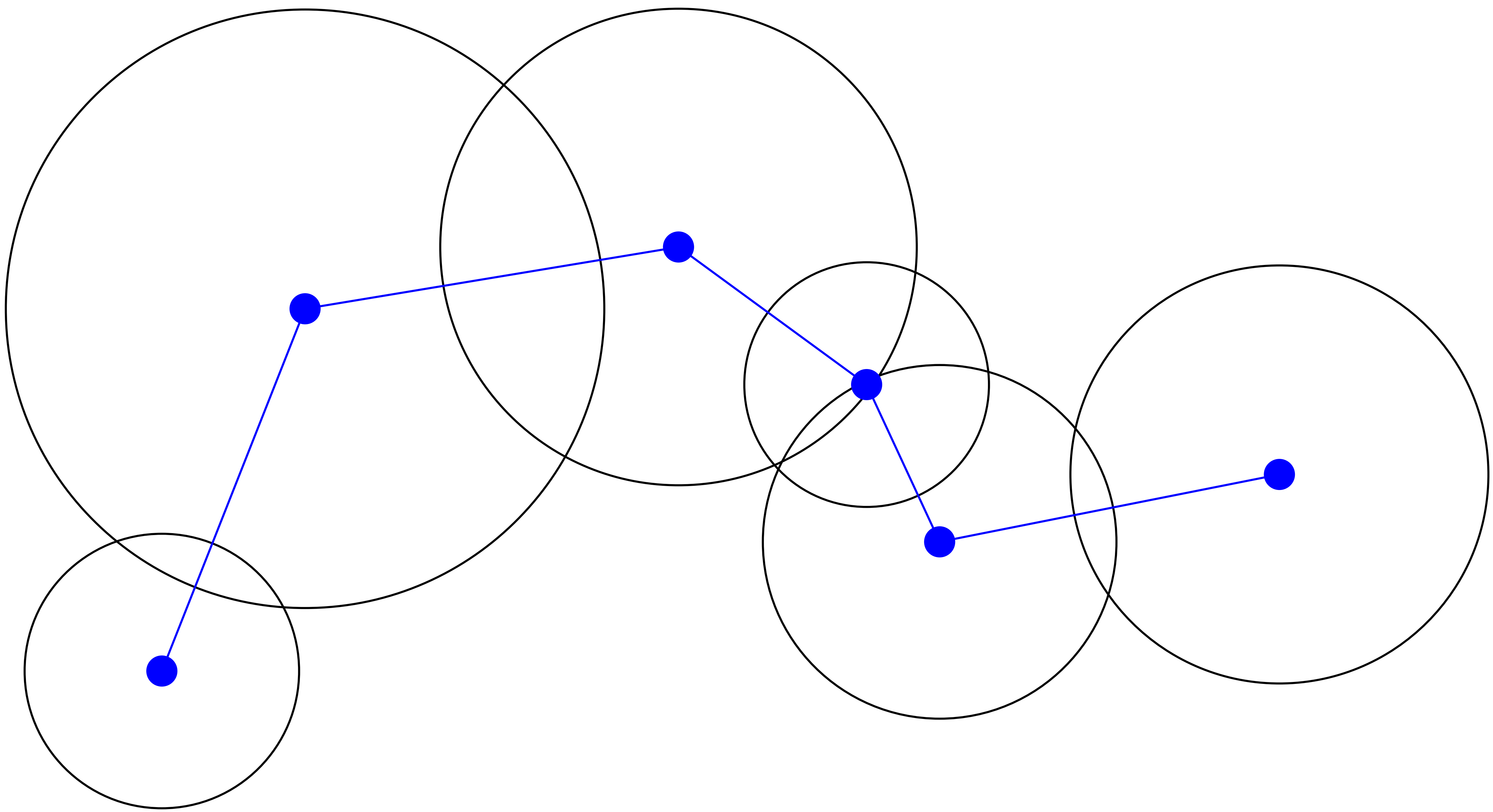

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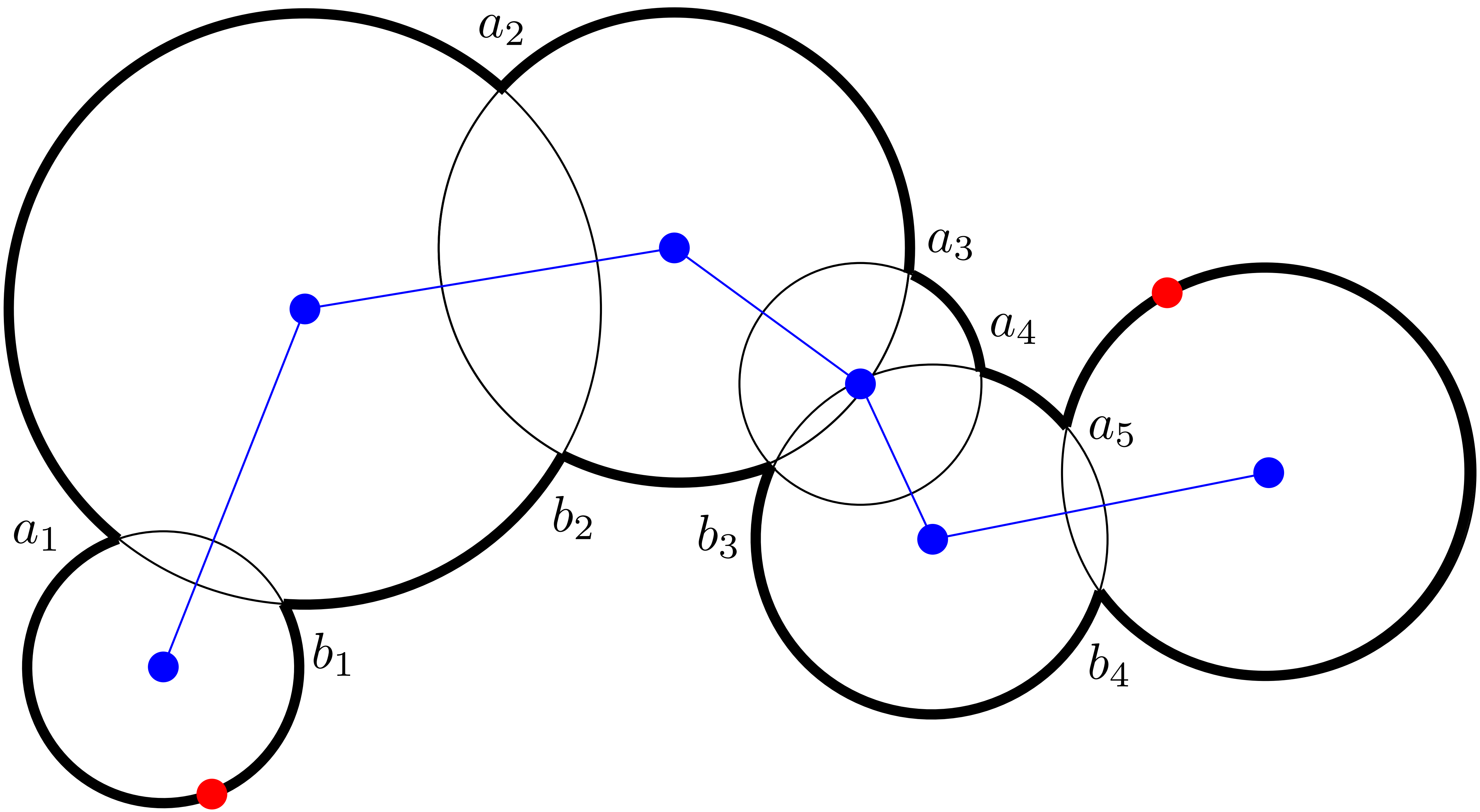

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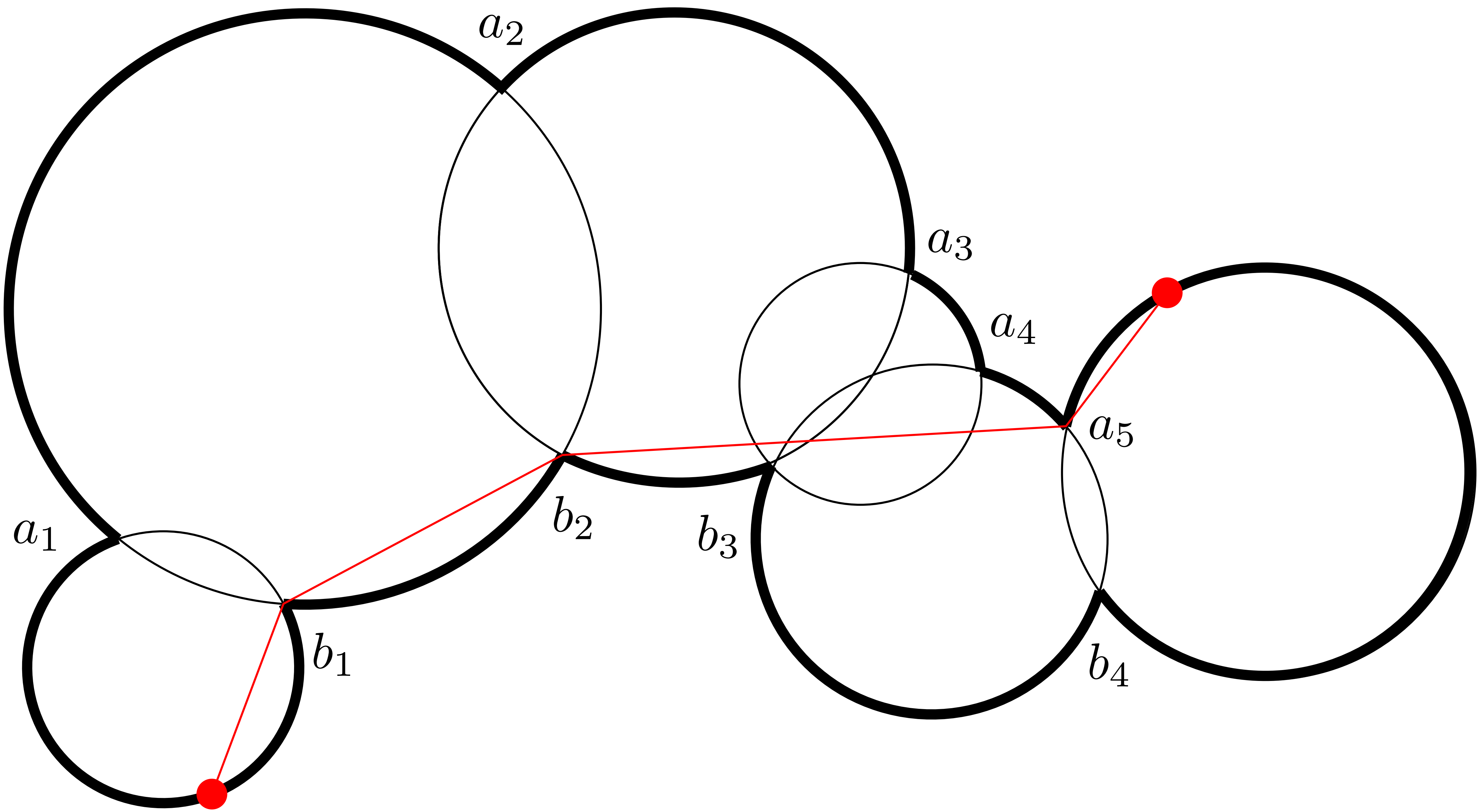

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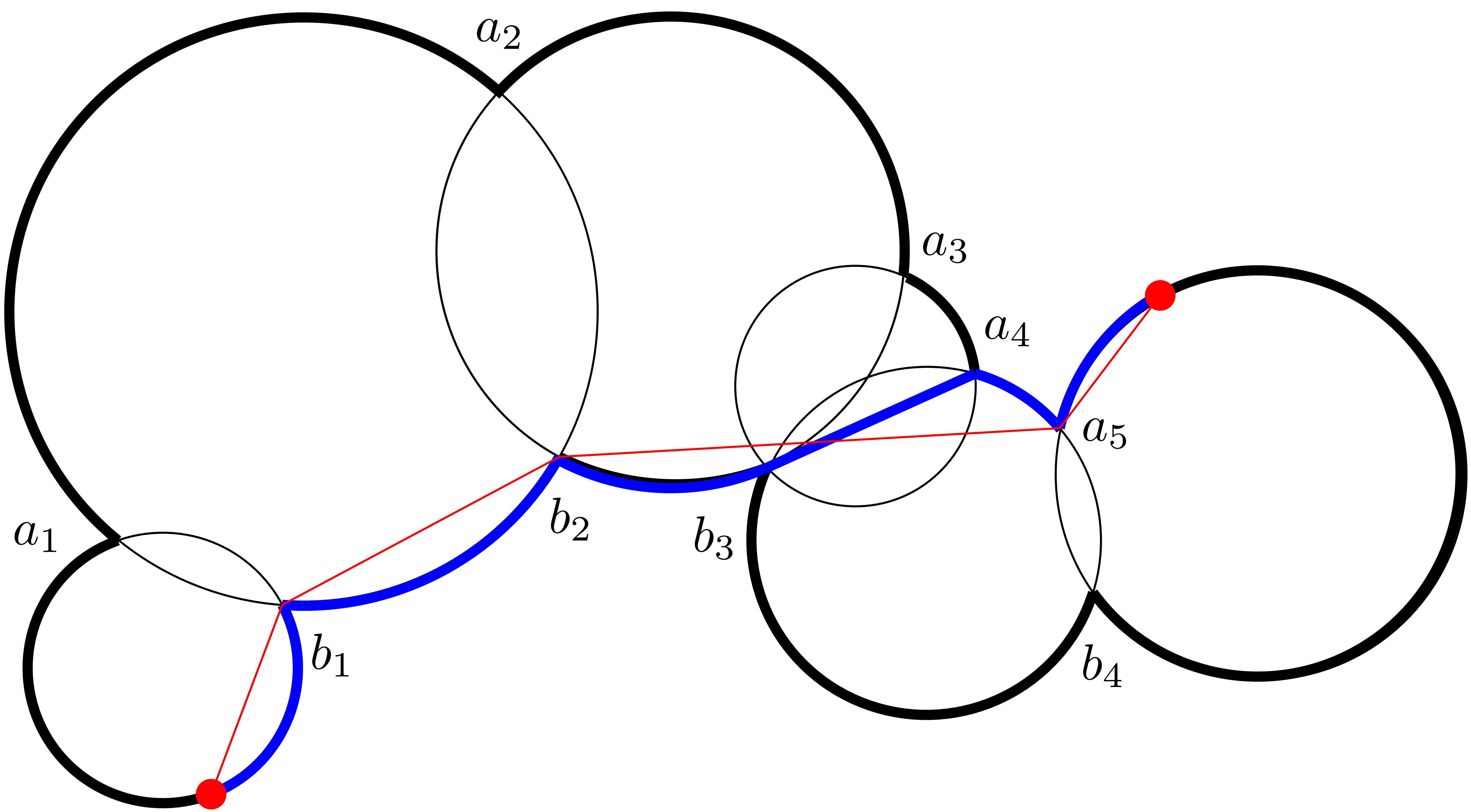

Standard Delaunay triangulation is a constant spanner:

Open Question:

What is the spanning ratio of the Delaunay Triangulation?

The best known upper bound on the spanning ratio is 1.998.

Standard Delaunay triangulation is a constant spanner:

What about routing on Delaunay graphs?

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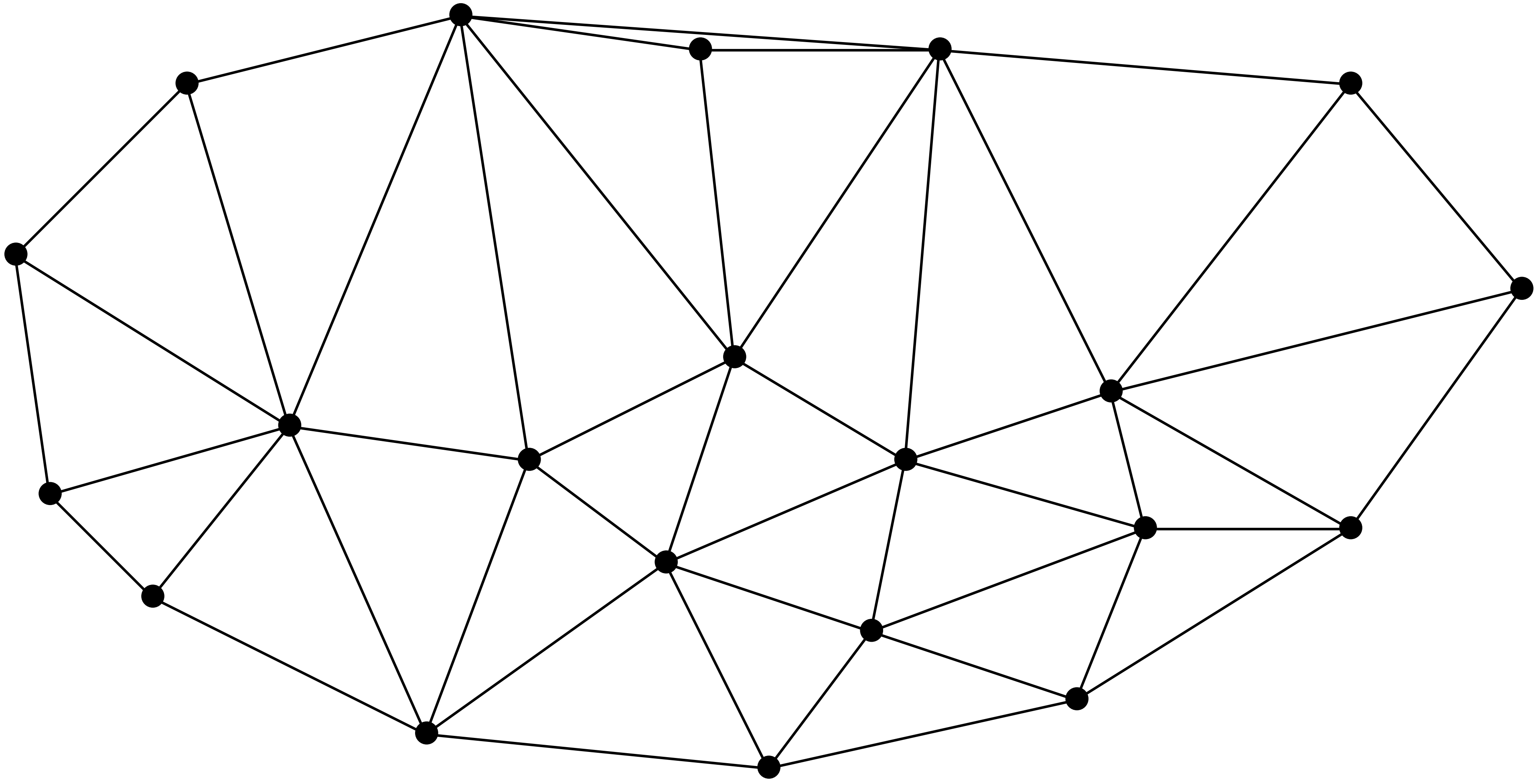

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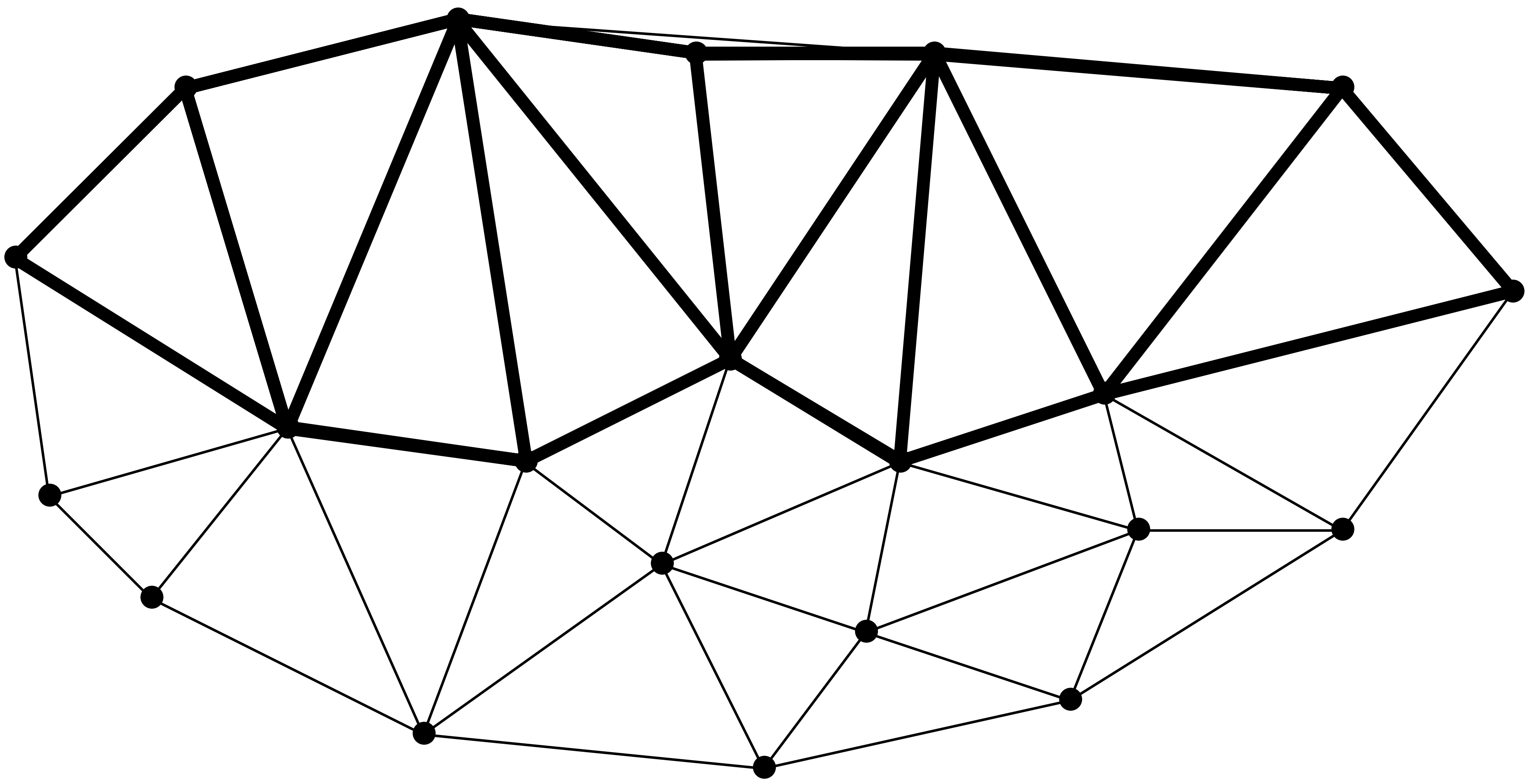

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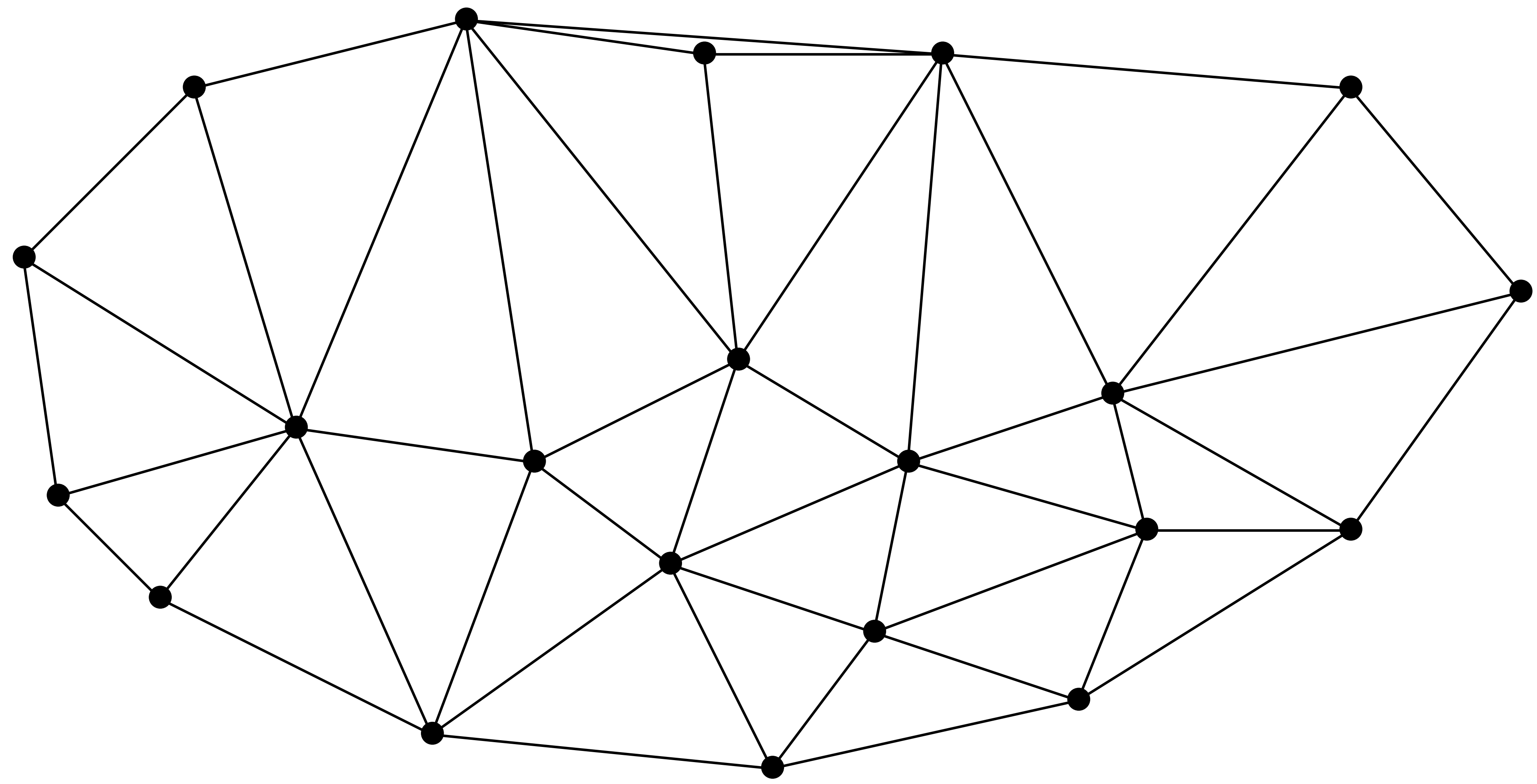

Standard Delaunay triangulation is a constant spanner:

What about lower bounds?

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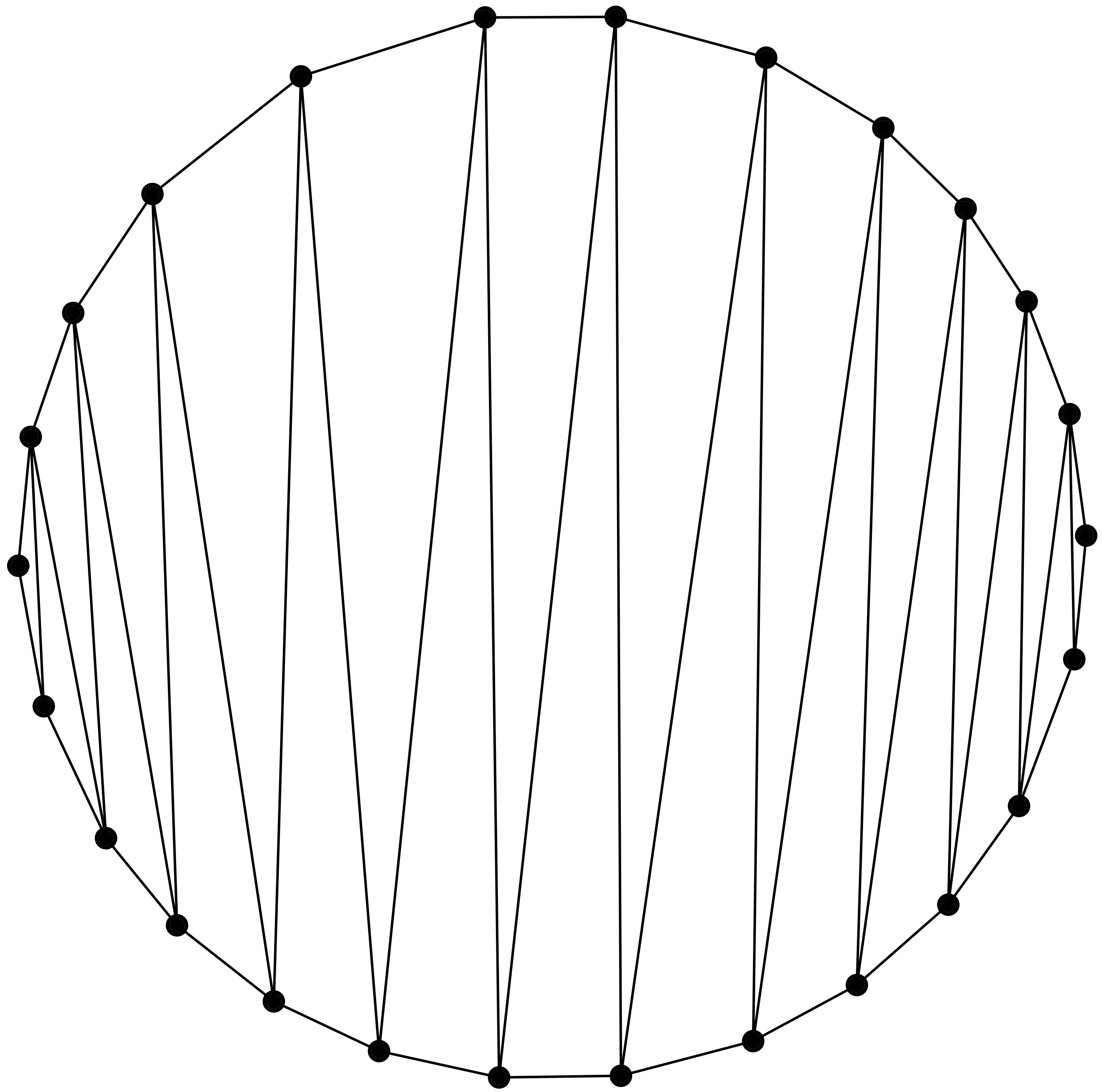


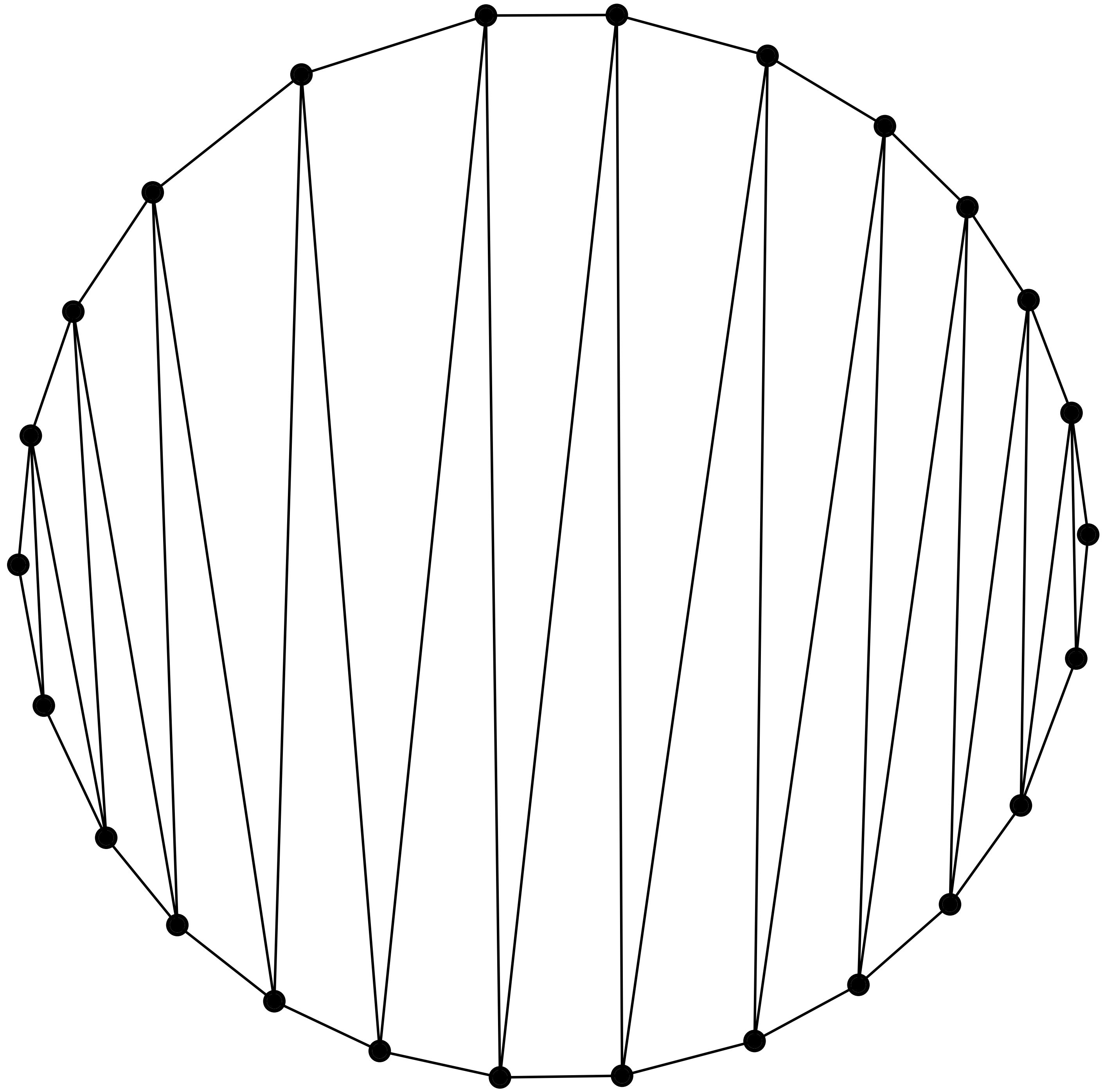
$\pi/2$ is a lower bound on the spanning ratio of Delaunay

Standard Delaunay triangulation is a constant spanner:

Is $\pi/2$ the right answer?

Standard Delaunay triangulation is a constant spanner:

B., Devroye, Loffler, Snoeyink, Verma (2009)



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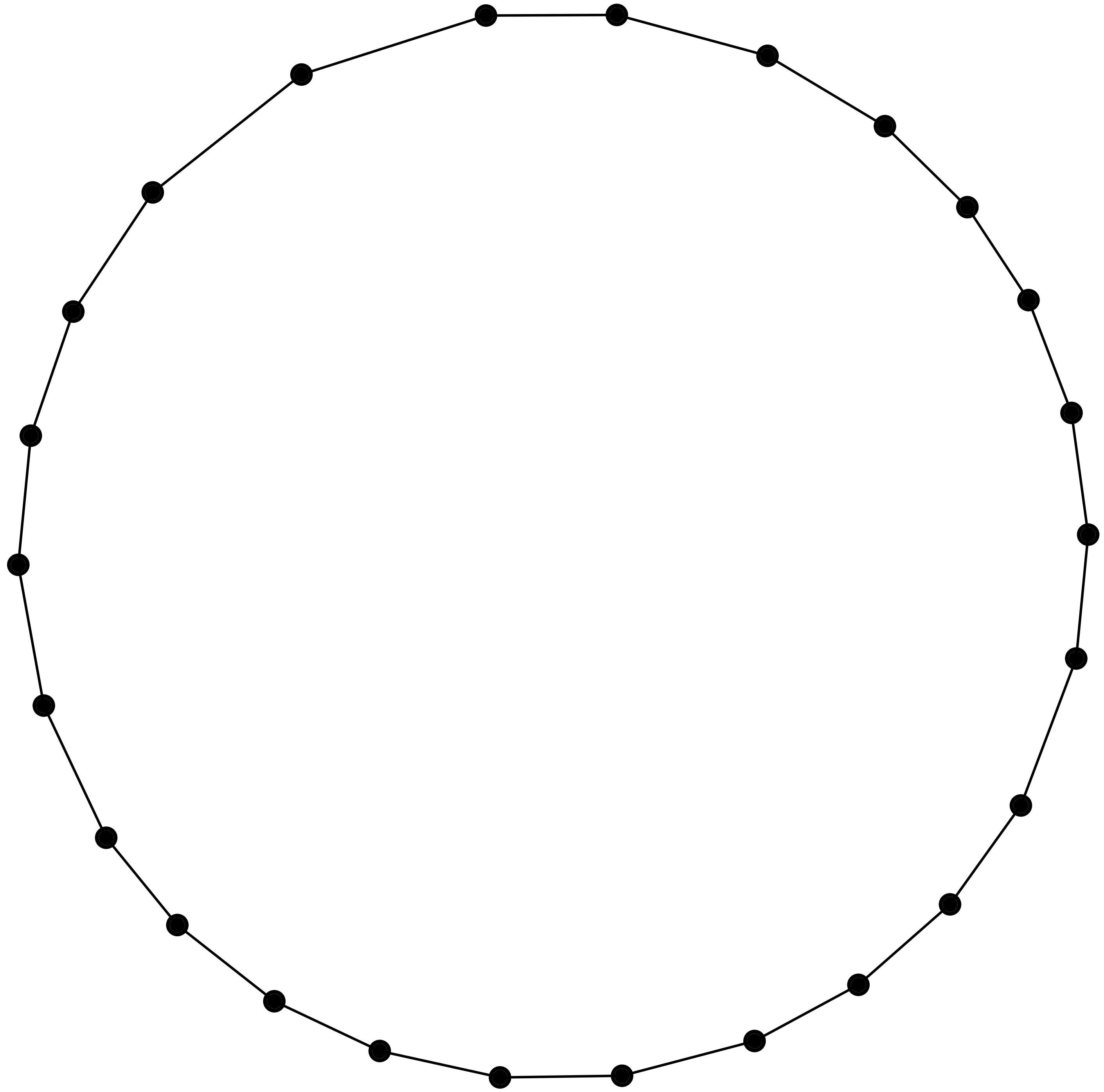

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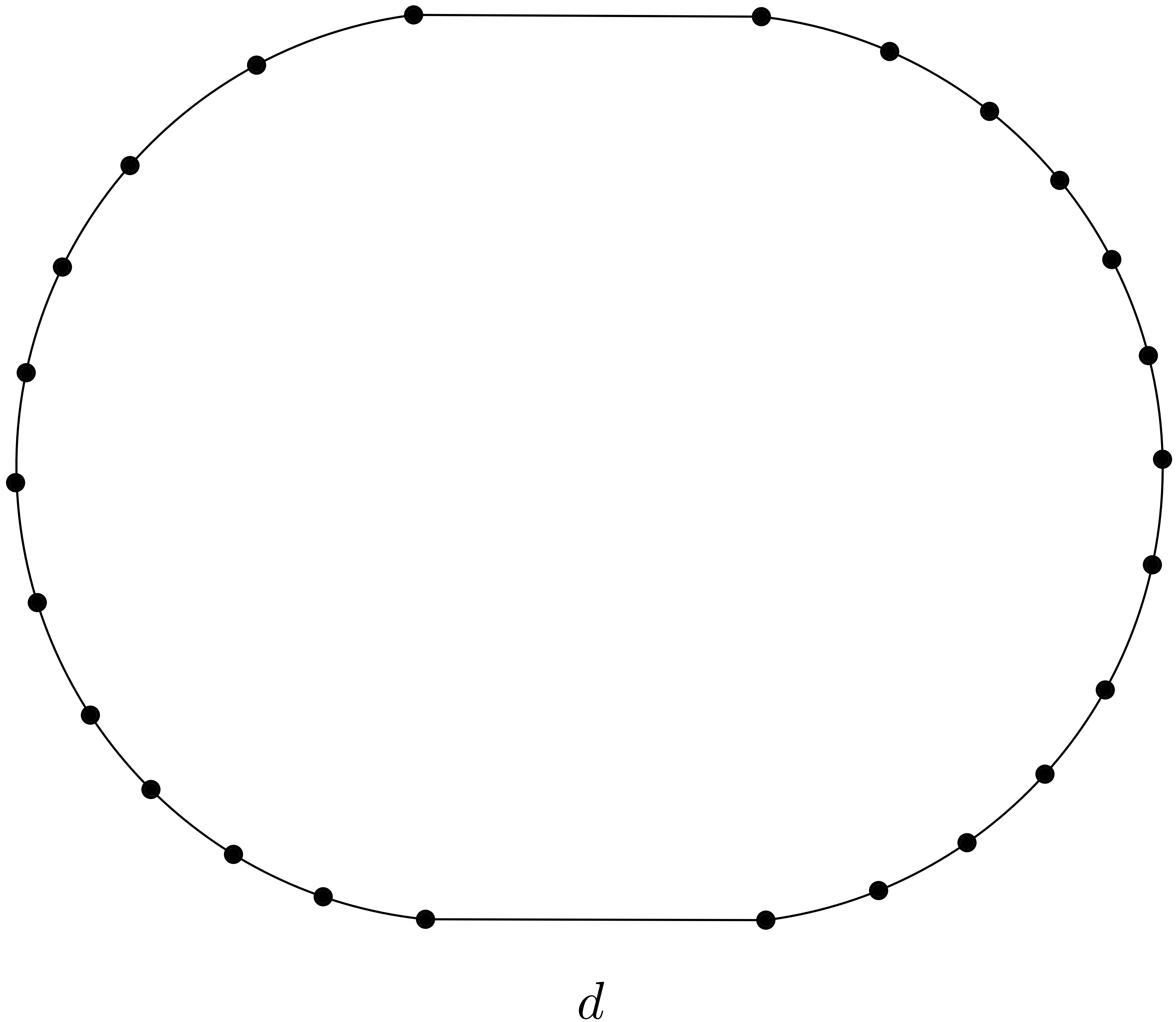

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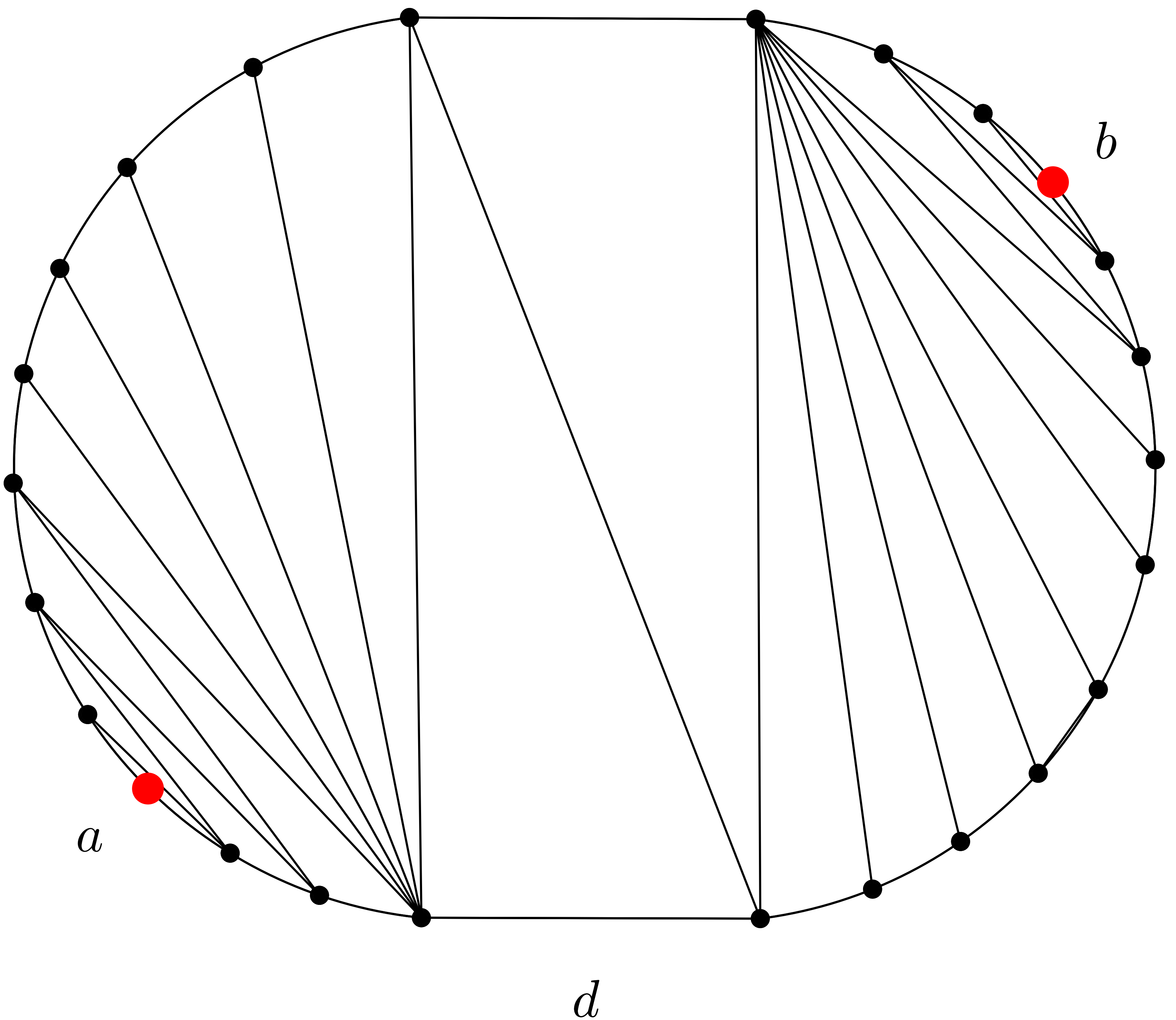

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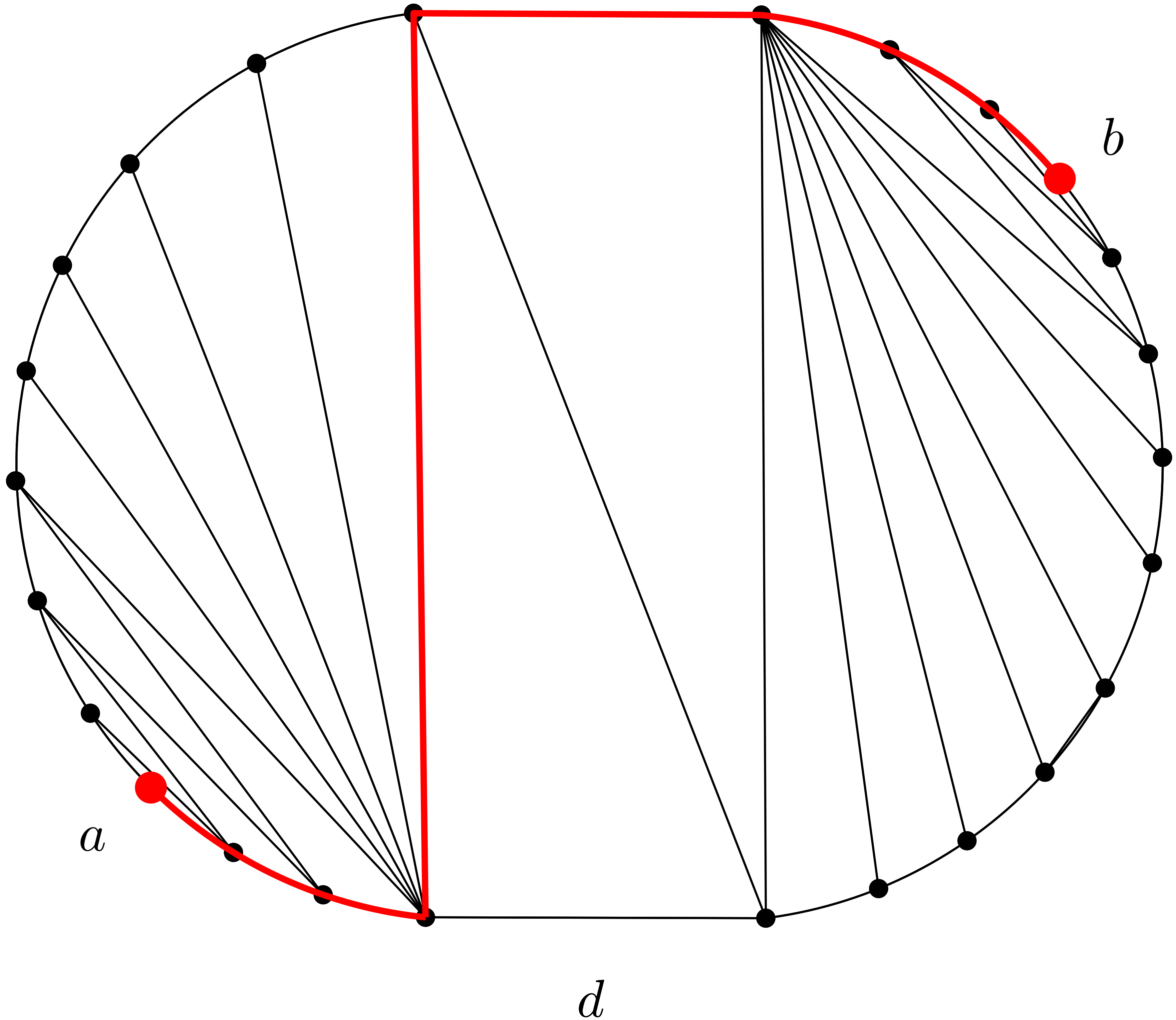

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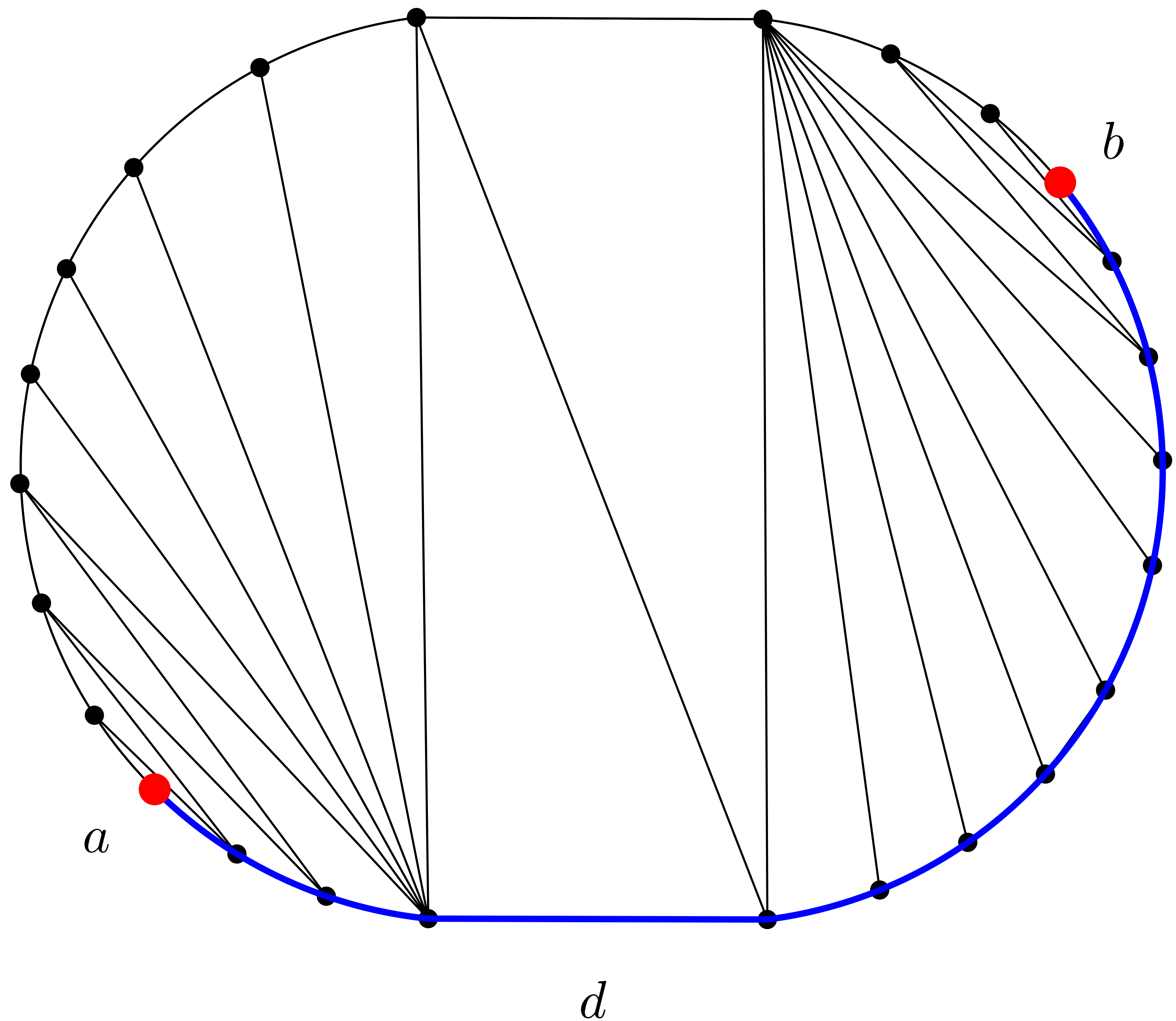

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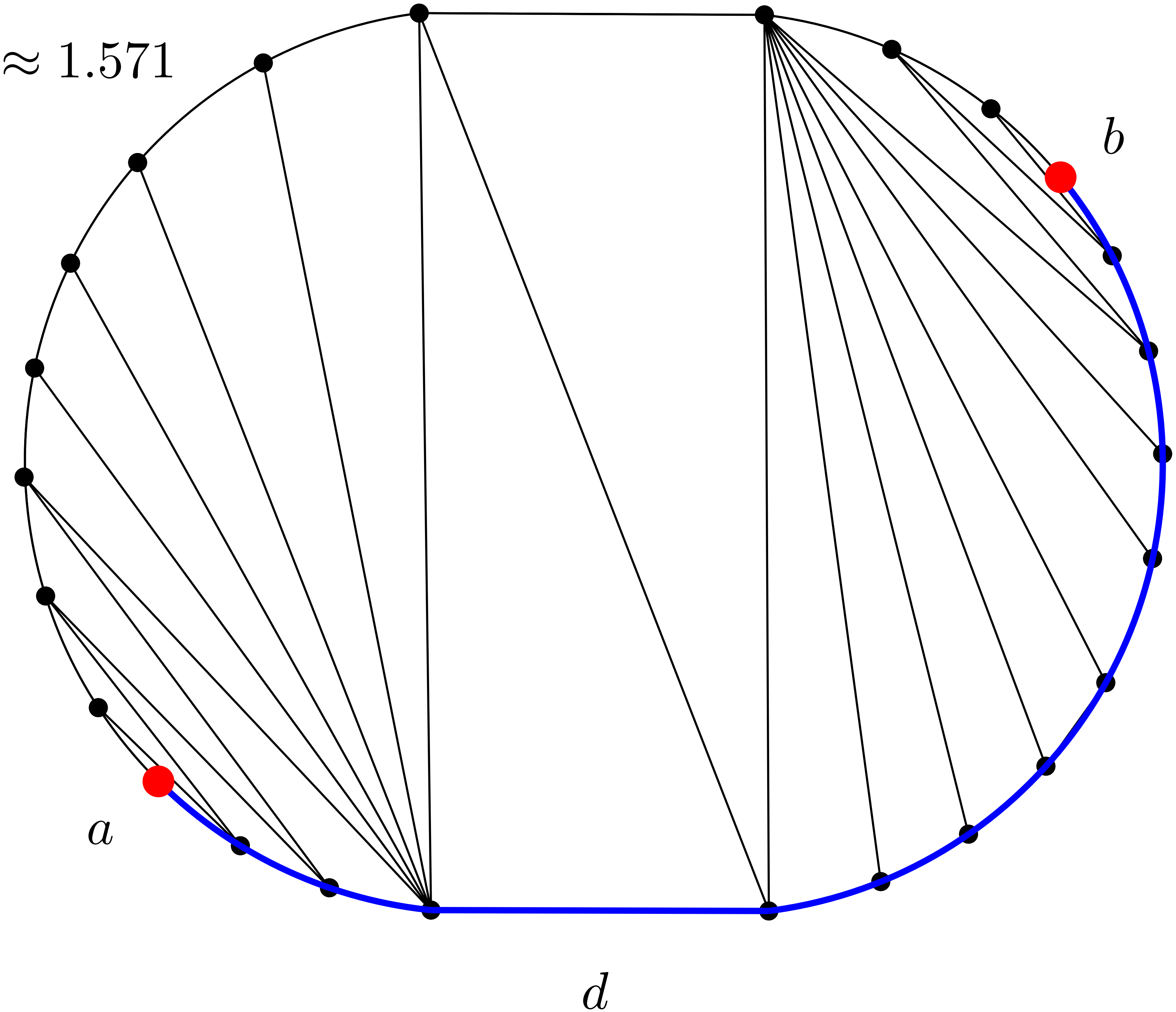
$1.581 > \pi/2 \approx 1.571$



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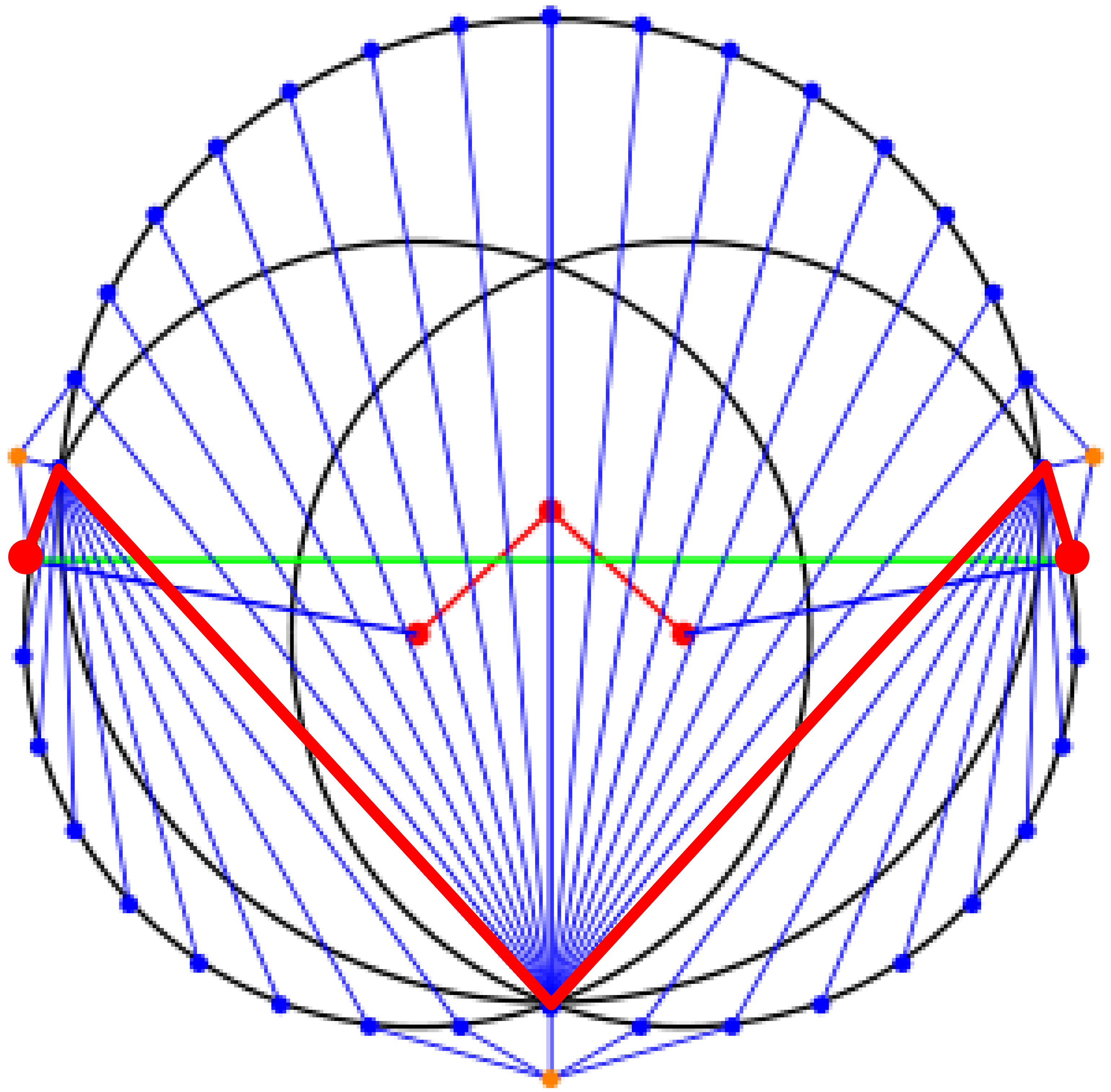

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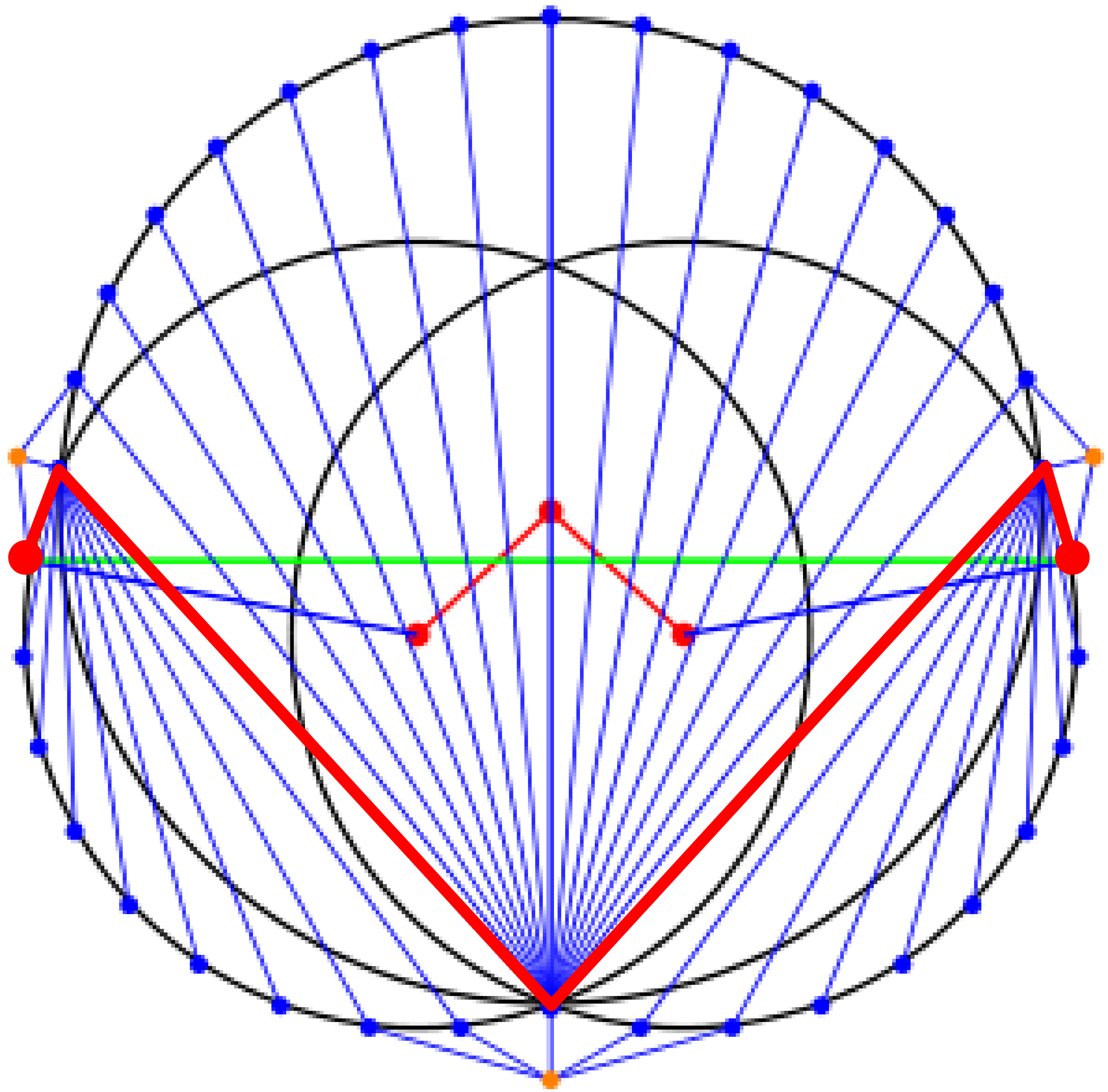
Lower Bound ≈ 1.593



Standard Delaunay triangulation is a constant spanner:

Open Question:

What is the spanning ratio of the Delaunay Triangulation?

The bound on the spanning ratio lies between 1.593 and 1.998.

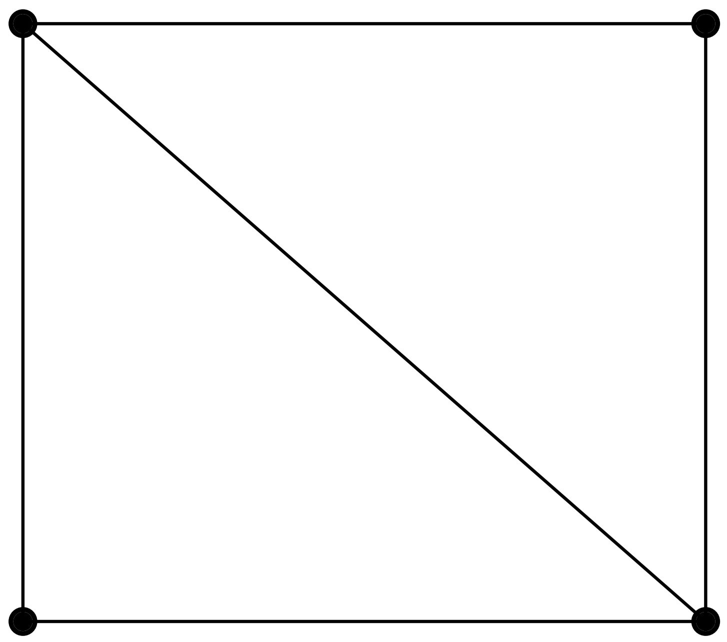
Plane Spanners:

Question:

What is a lower bound on the spanning ratio that a planar graph can achieve on every point set?

Lower Bounds:

$\sqrt{2}$ is a trivial lower bound.



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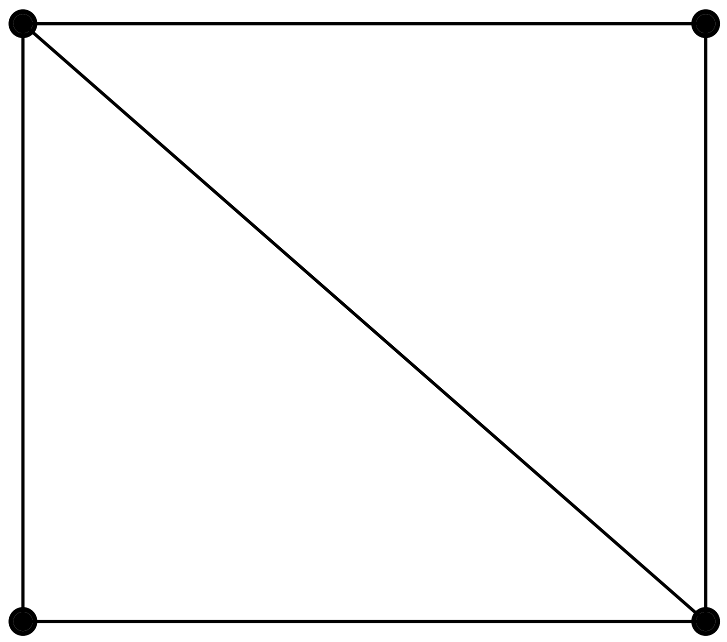

Can we do better?

Mulzer (2004) showed that every triangulation of a regular 21-gon has spanning ratio at least $1.41611 \approx \sqrt{2.005367532}$

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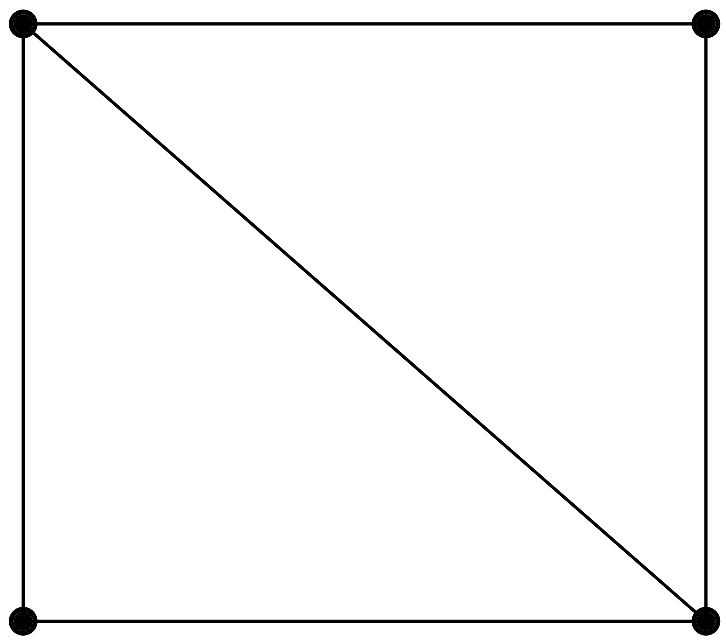

Regular 15-gon: 1.40898
Regular 17-gon: 1.40875
Regular 18-gon: 1.38170

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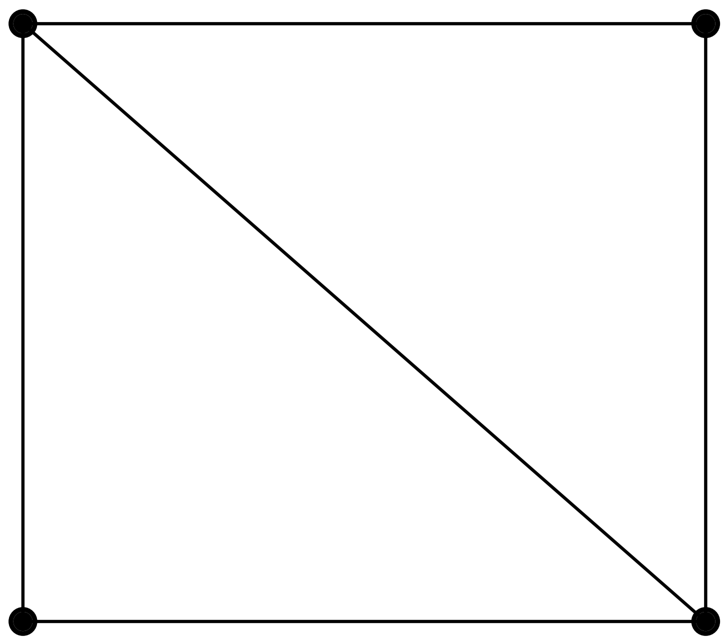


Can we do better?

Every triangulation of a regular 23-gon has spanning ratio at least 1.4308 [Dumitrescu and Ghosh (2018)]

Open Question:

What is the best lower bound for the spanning ratio of a triangulation?

Specifically, is there a $t > 1.4308$ such that given a point set P, every triangulation of P has spanning ratio at least t?

Recall that the lower bound for Delaunay triangulations is 1.593.

The spanning ratio of plane spanners lies between 1.4308 and 1.998

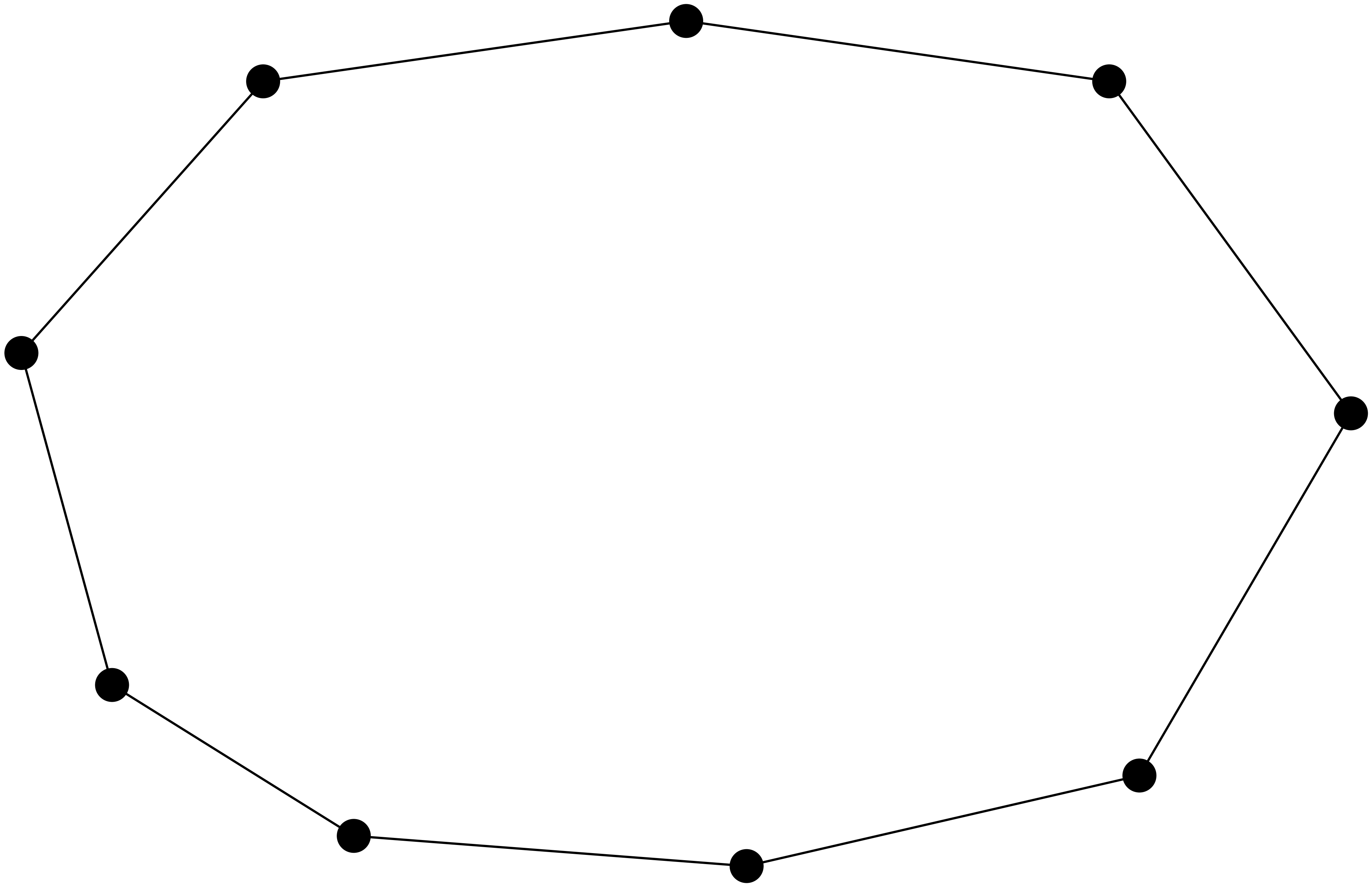
Question:

Can we do better if the points are in convex position?

Points in Convex Position:

Amani et al. (2016):

Every set of points in convex position has a linear size spanner whose spanning ratio is 1.88.



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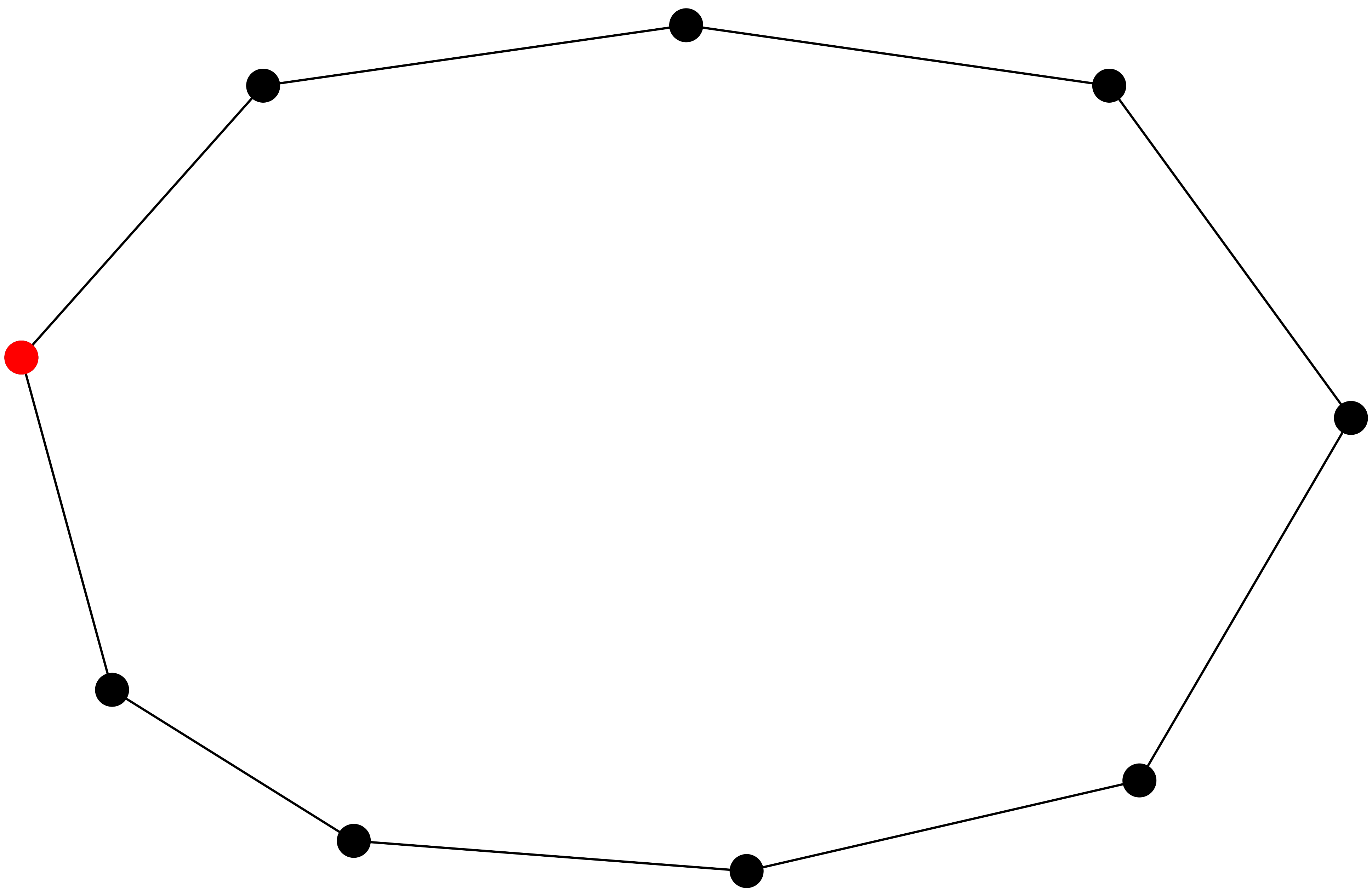

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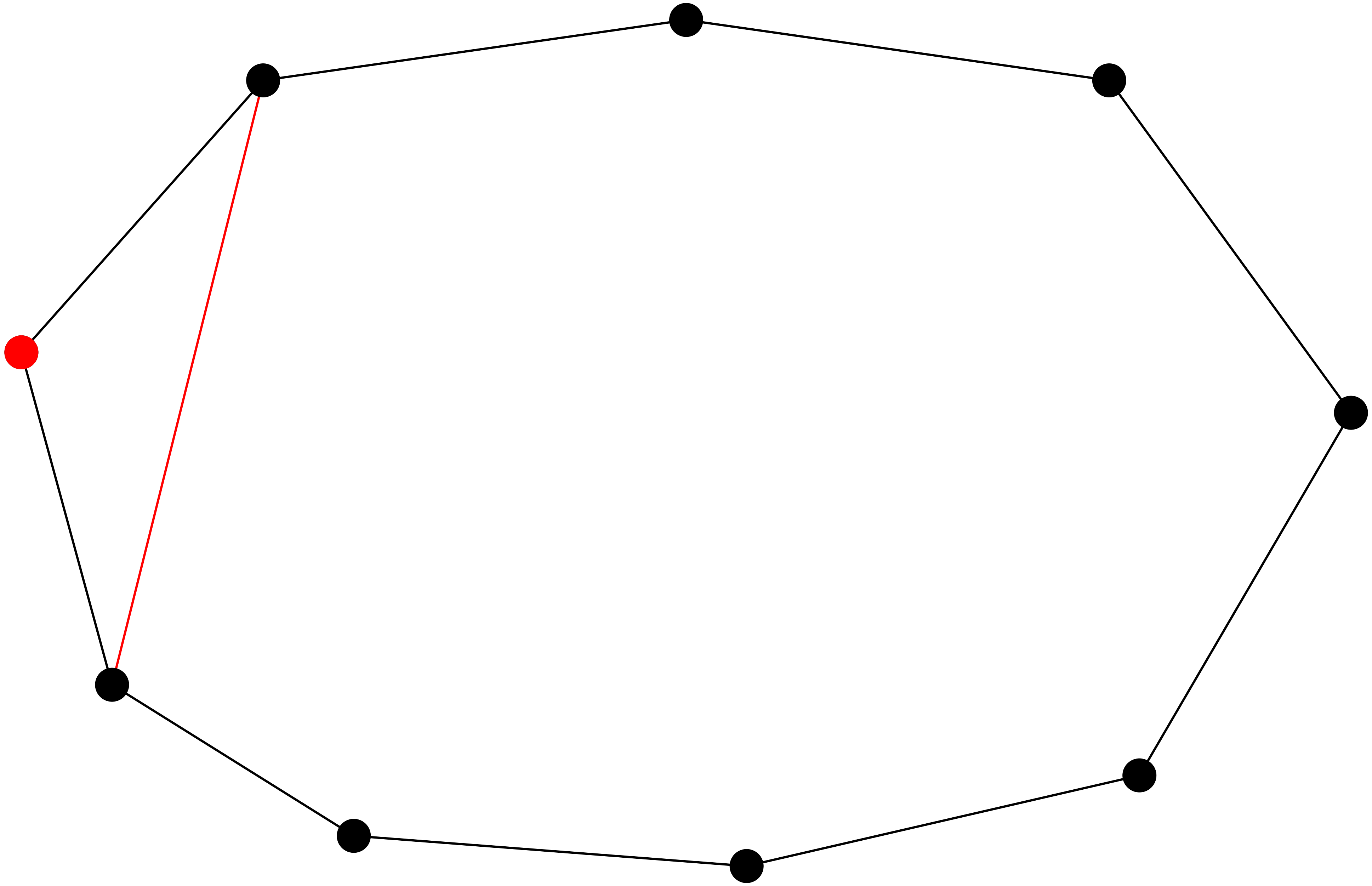

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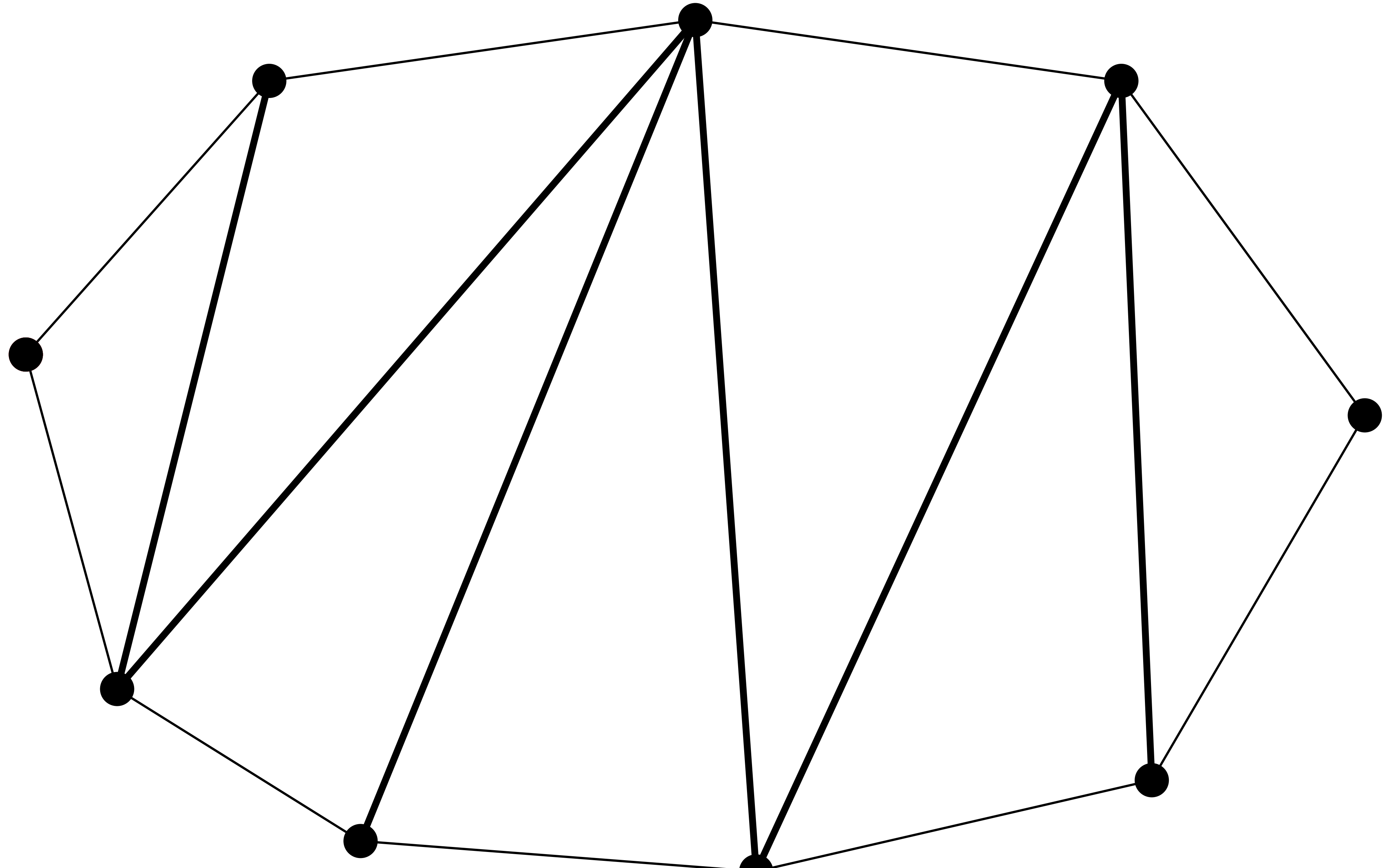

Points in Convex Position:

Tan et al. (2019) showed that the Delaunay triangulation of a set of points in convex position has spanning ratio at most 1.83.

Open Question:

What is the best spanning ratio for points in convex position?

The spanning ratio of plane spanners of points lying in convex position is some value between 1.4308 and 1.83 and for Delaunay triangulations, it is between 1.581 and 1.83.

Question:

Are Delaunay graphs still constant spanners if we change the empty region from a disk to something else?

Delaunay Graphs with other types of empty regions:

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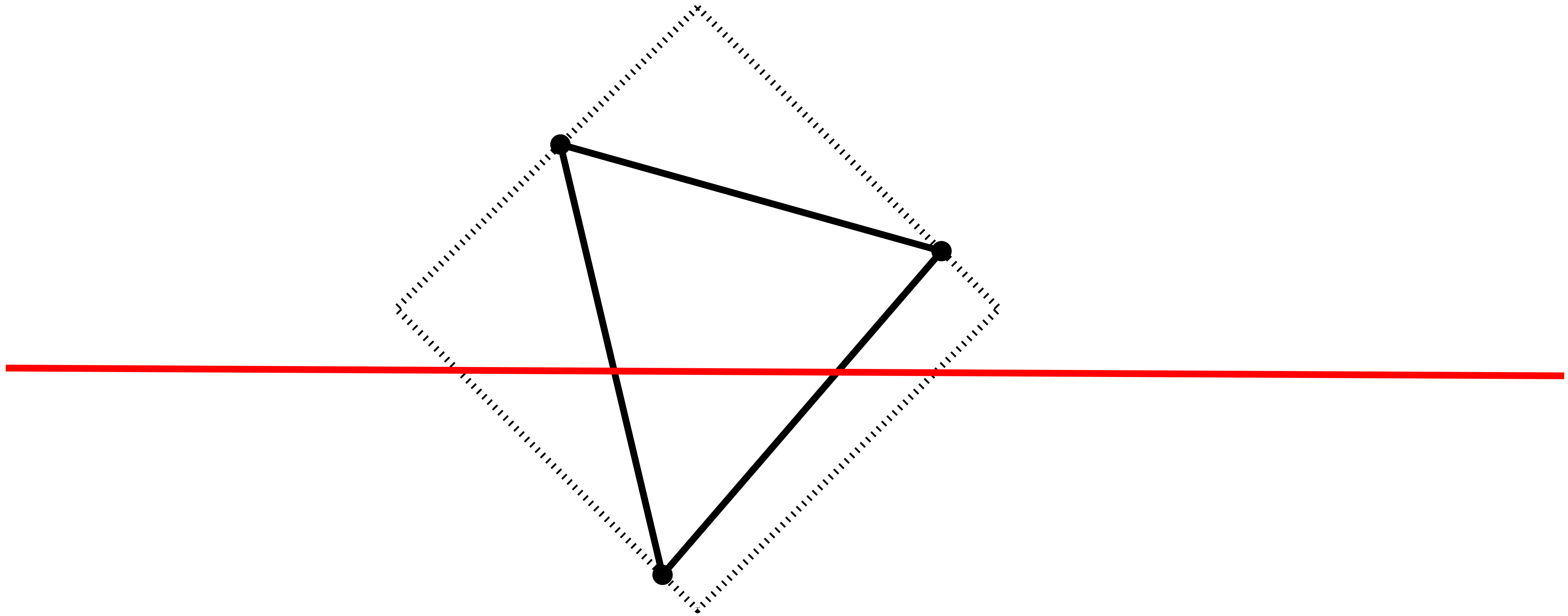
Proof Idea:



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Denis, Perković and Turkoglu (2021) showed that the empty hexagon Delaunay is a 2-spanner

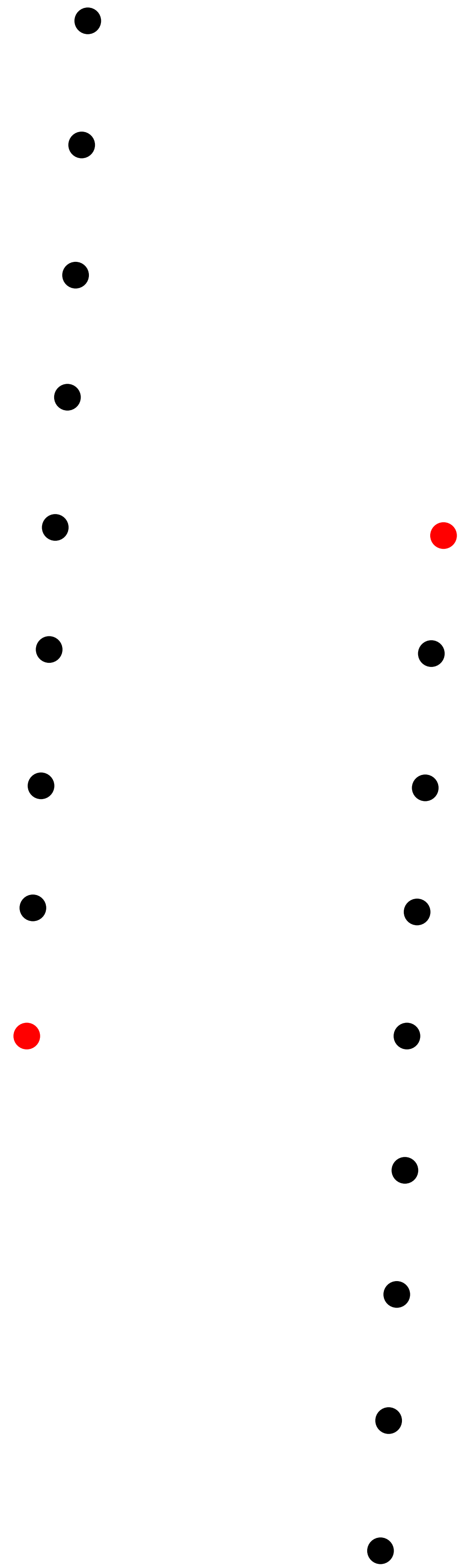
Delaunay Graphs with other types of empty regions:

C	Spanning Ratio	Routing Ratio
Triangle	2 [Chew 89]	$5/\sqrt{3}$ [B. et al 15]
Square	$\sqrt{4+2\sqrt{2}}$ [Bonichon et al. 15]	$\sqrt{10}$ [Chew 89]
Pentagon	≈ 4.640 *	≈ 4.640*
Hexagon	2 [Perkovic et al. 21]	≈ 6.429*
Septagon	≈ 8.53 *	≈ 8.53*
Octagon	≈ 4.05 *	≈ 4.05*
Circle	≈ 1.998 [Xia 13]	≈ 3.56 [B. et al. 24]

*B., Carufel, Stuart 24

Delaunay Graphs with other types of empty regions:

Lower bound construction:



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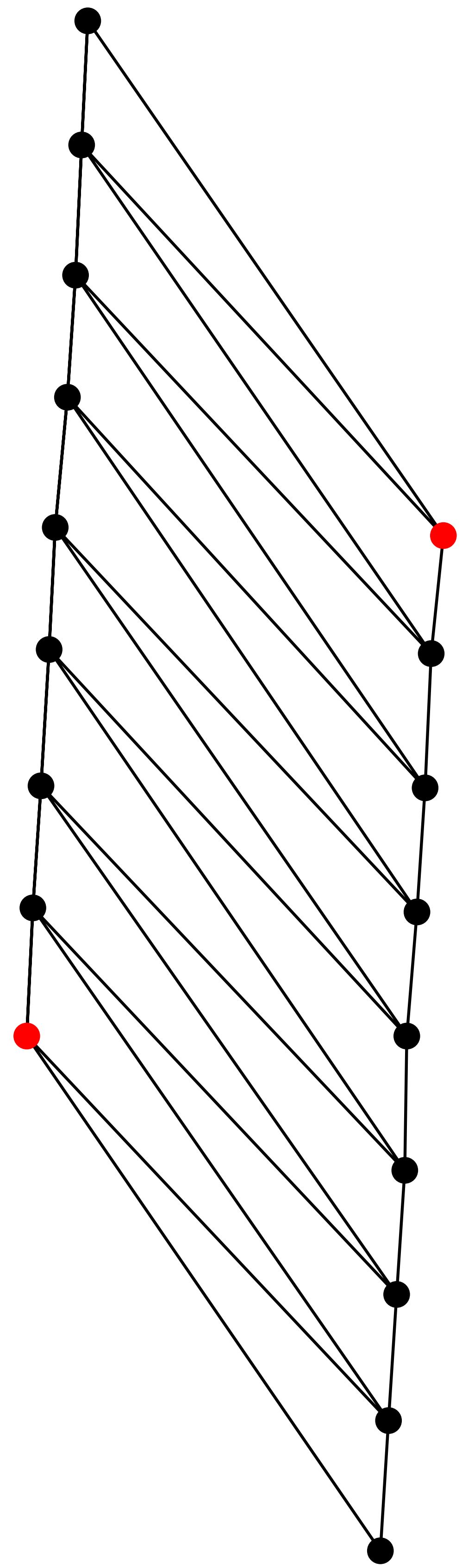
Lower bound construction:

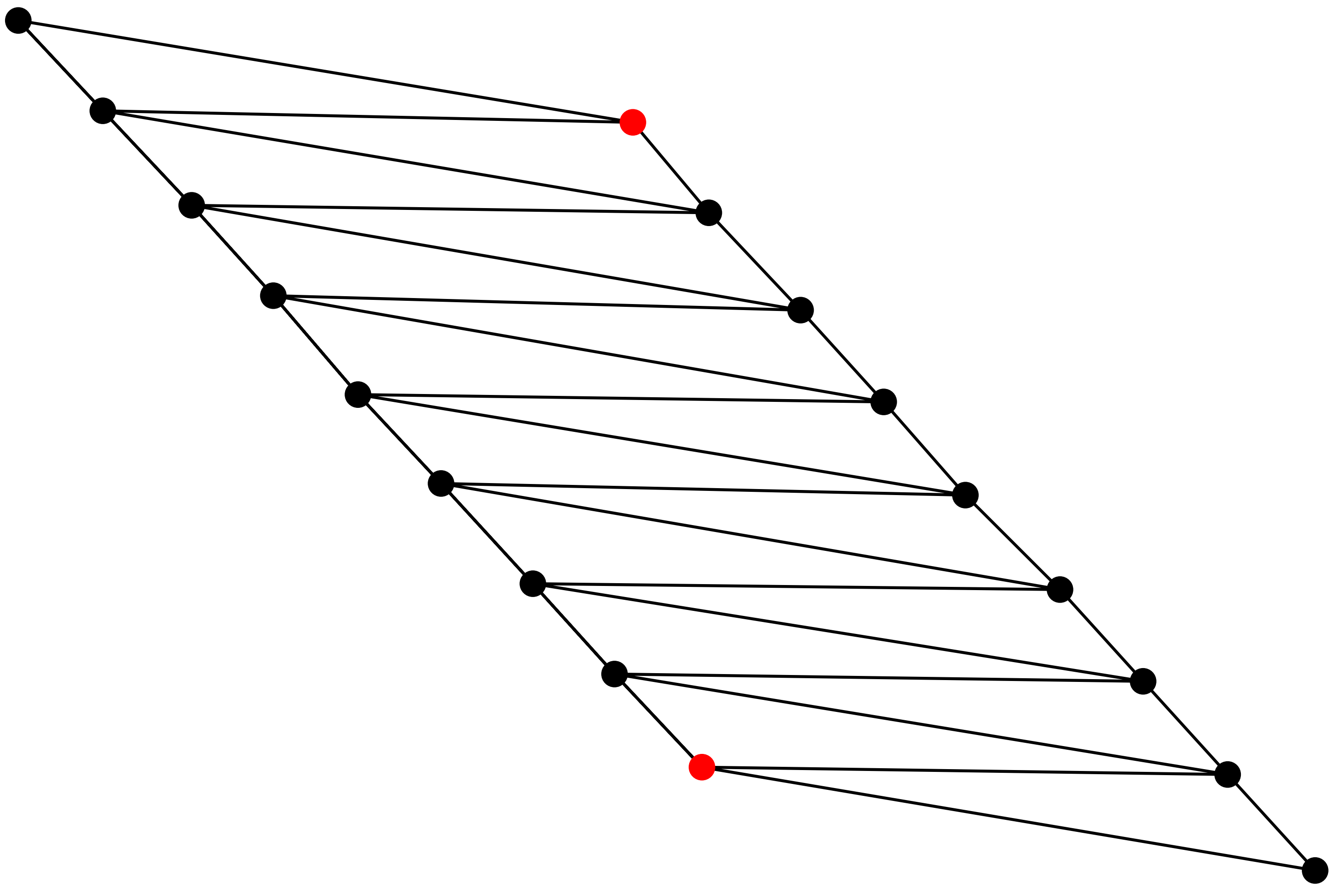
Spanning ratio ≈ 2.61



Delaunay Graphs with other types of empty regions:

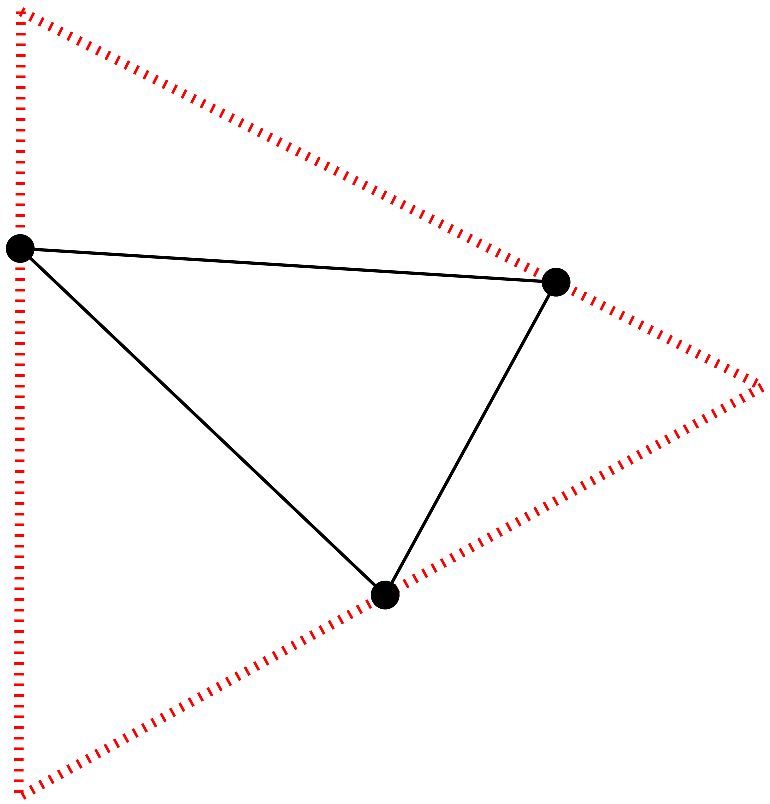
Dennis, Perkovic and Turkoglu (2020) showed a spanning ratio of 2 for Empty Hexagon Delaunay.

Their lower bound construction is similar to that of squares.



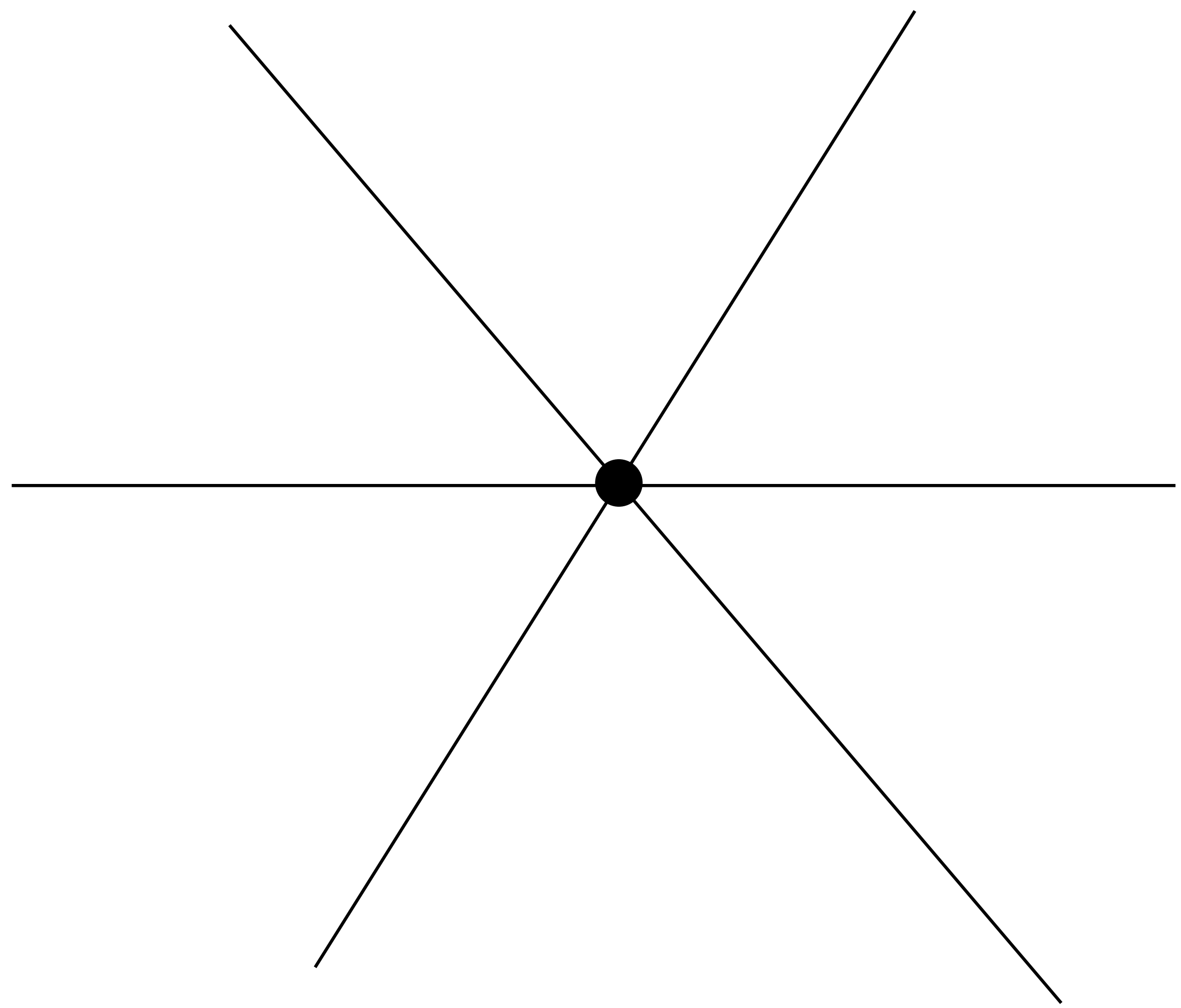
Delaunay Graphs with other types of empty regions:

When the empty region is an equilateral triangle, Chew (1989) showed a spanning ratio of 2.



Connection between $\triangle$-Delaunay and θ-graphs

Bonichon, Gavoille, Hannuse, Ilcinkas [2010]



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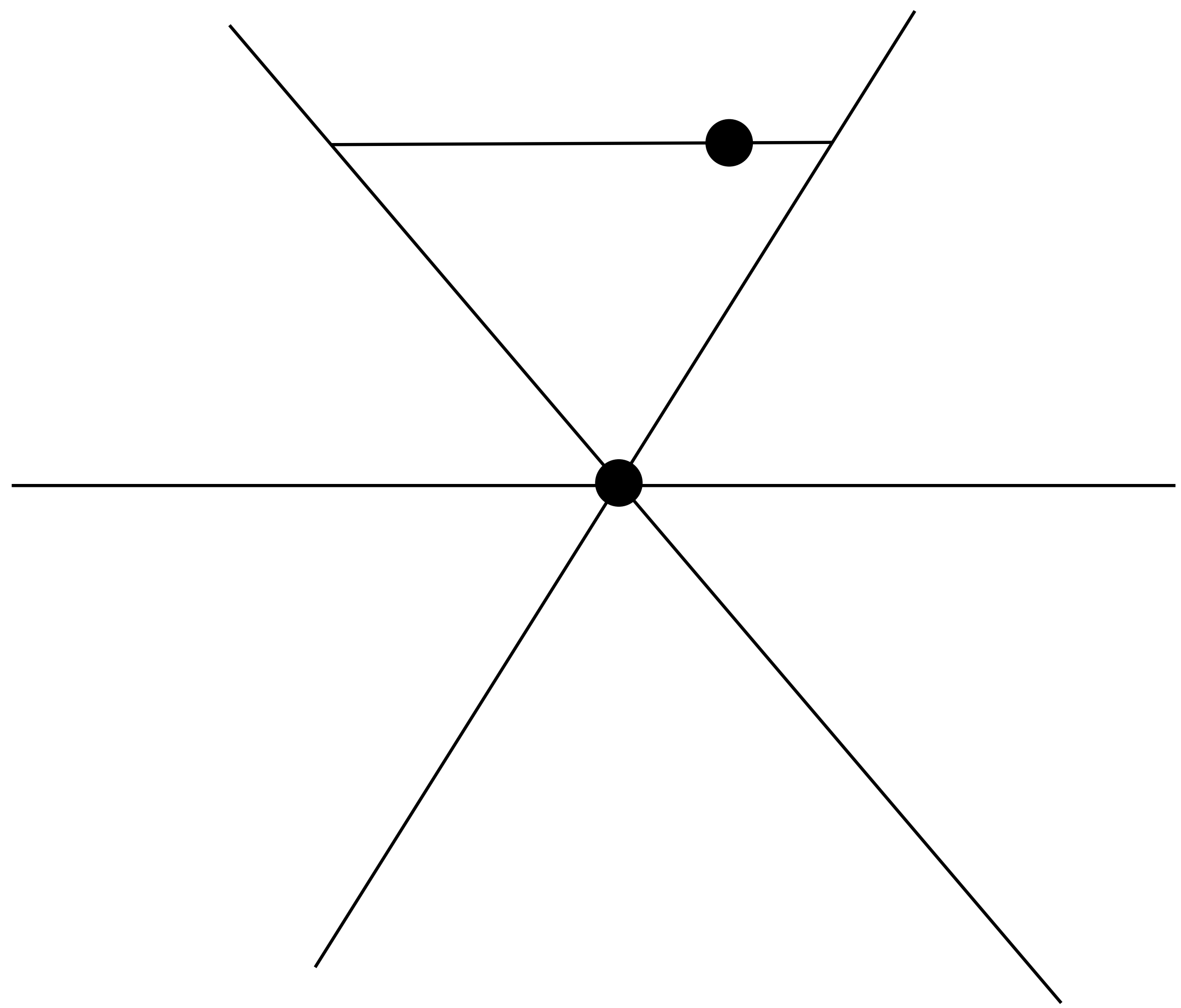

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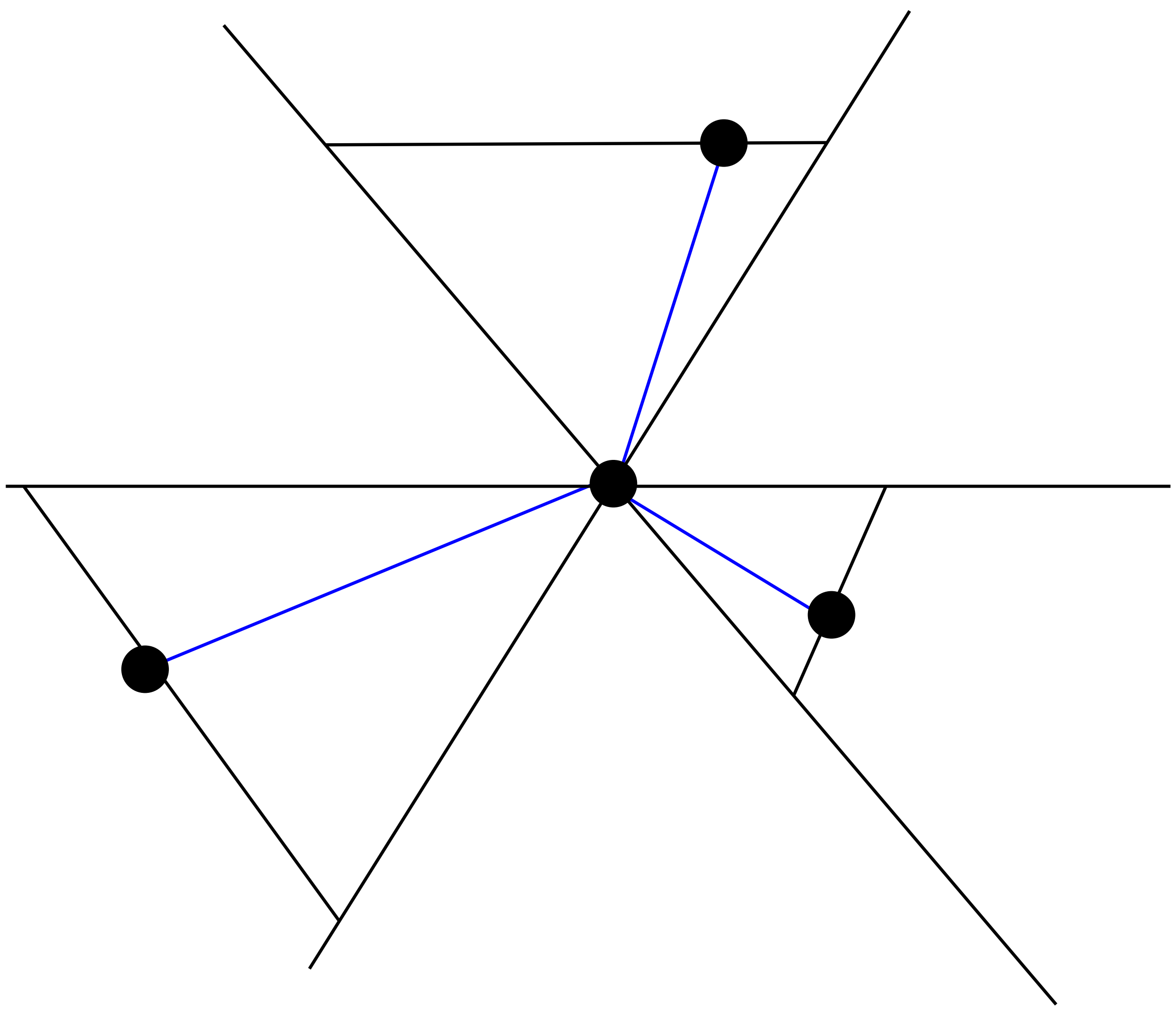

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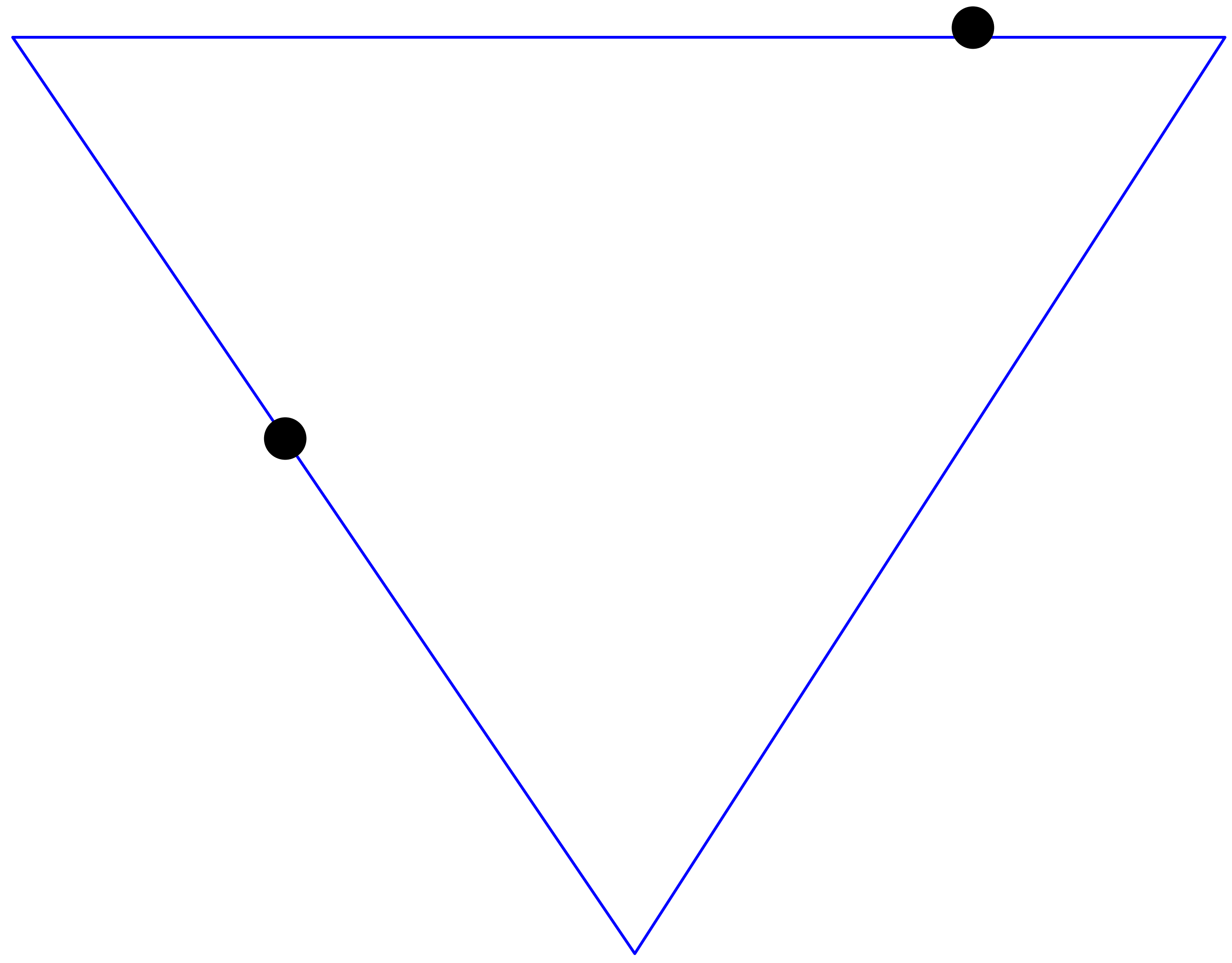

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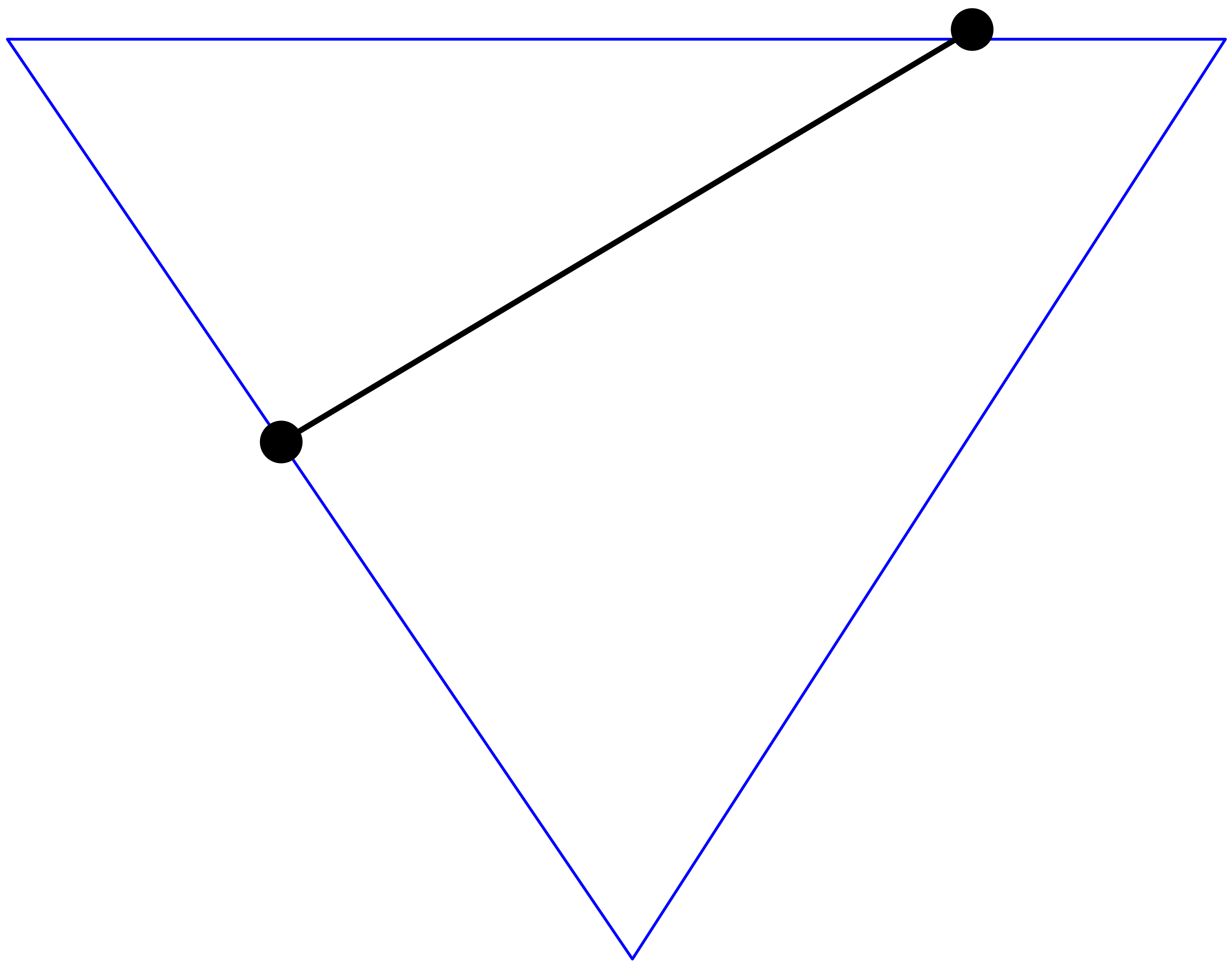

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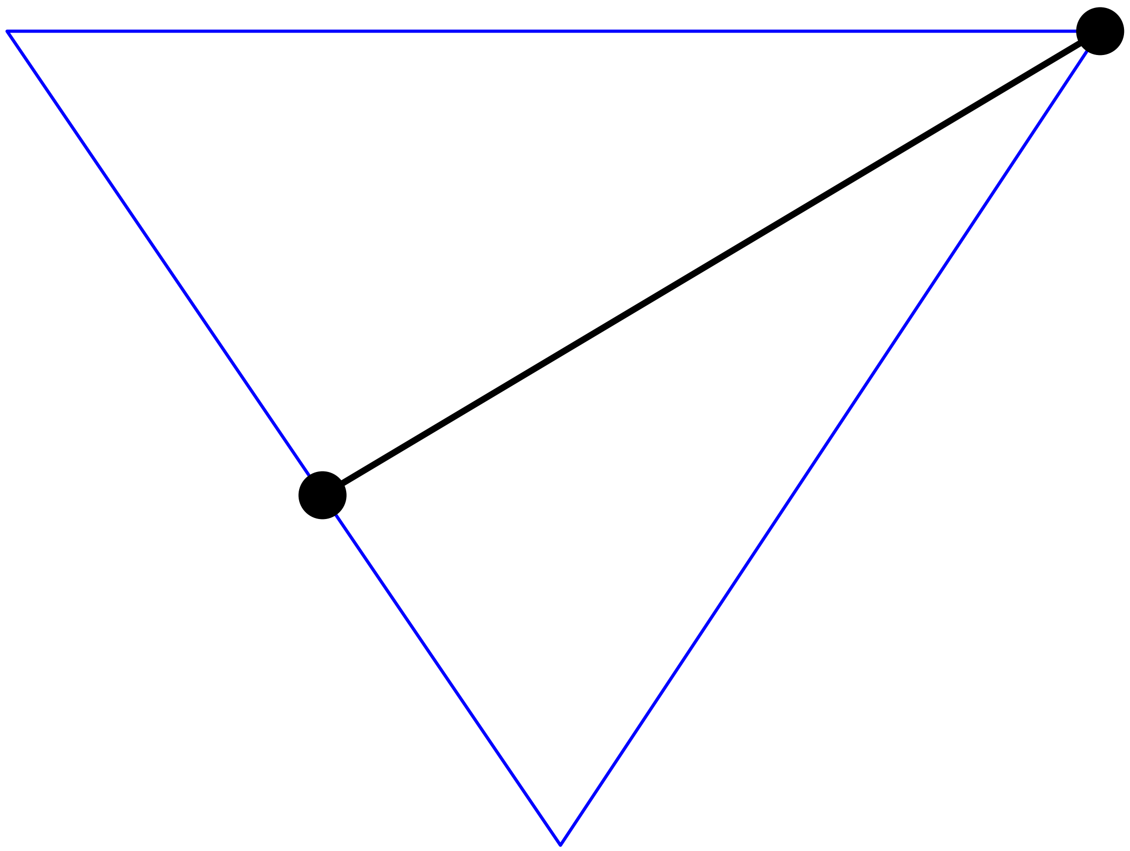

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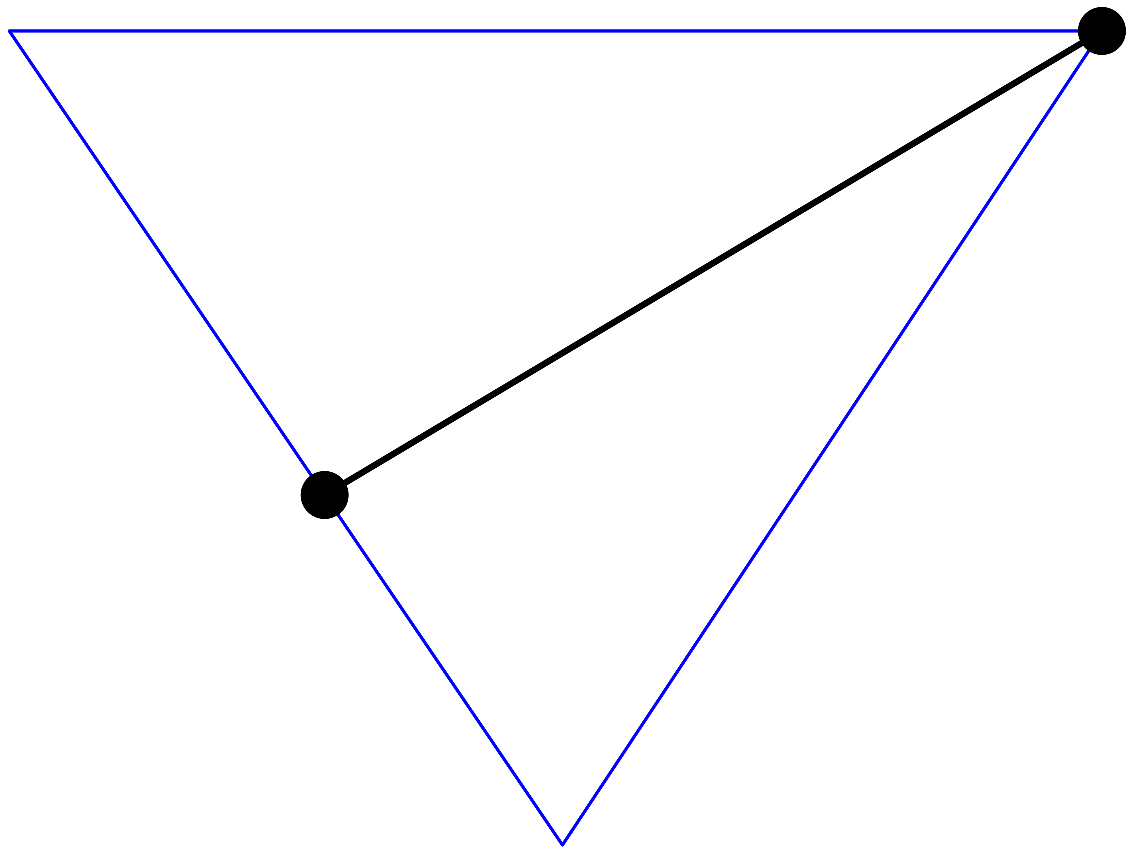
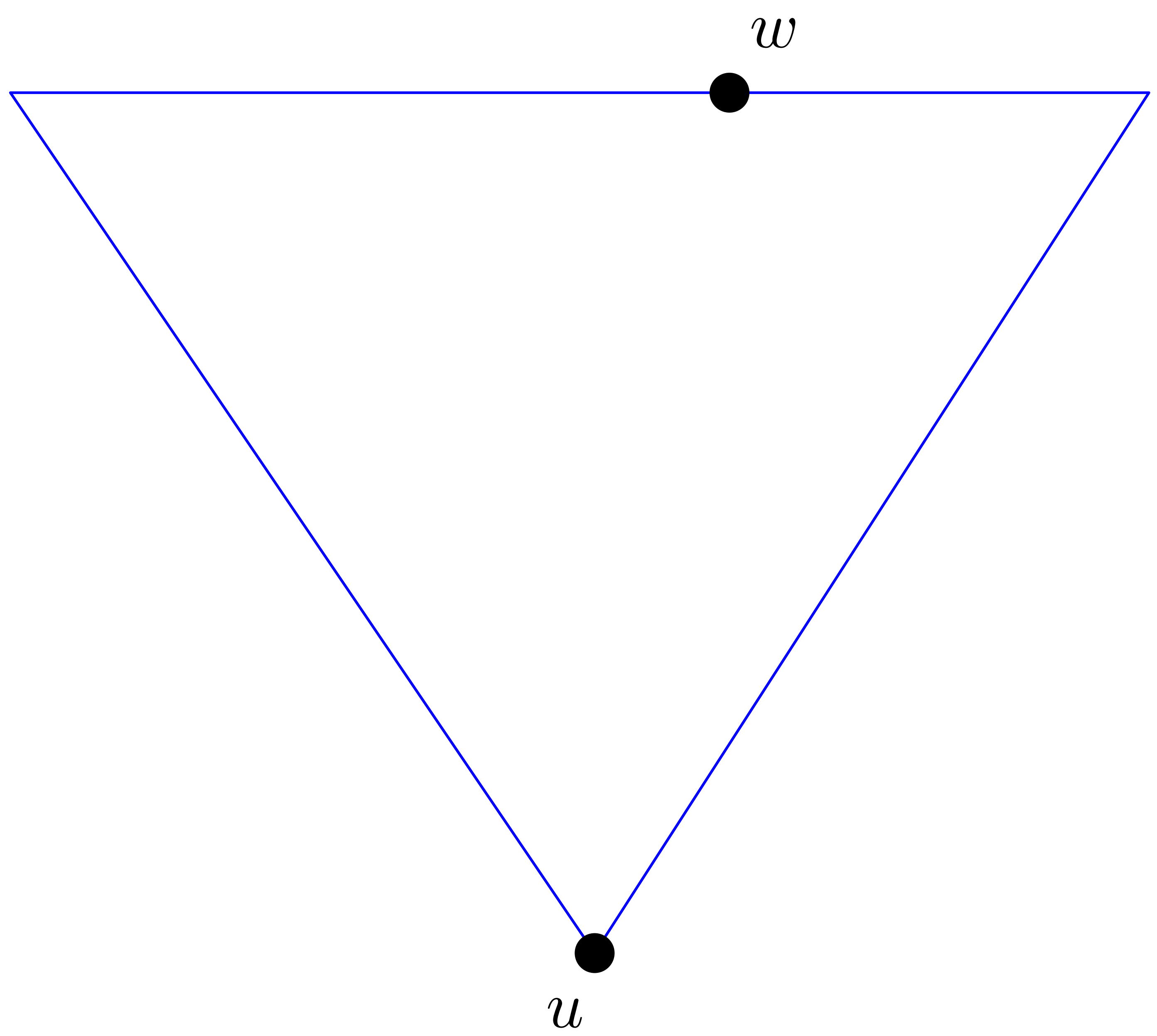


There is a 1-1 correspondence between half-θ and $\triangle$-Delaunay

Connection between △-Delaunay and θ-graphs

B., Fagerburg, van Ressen, Verdenschot [2011]



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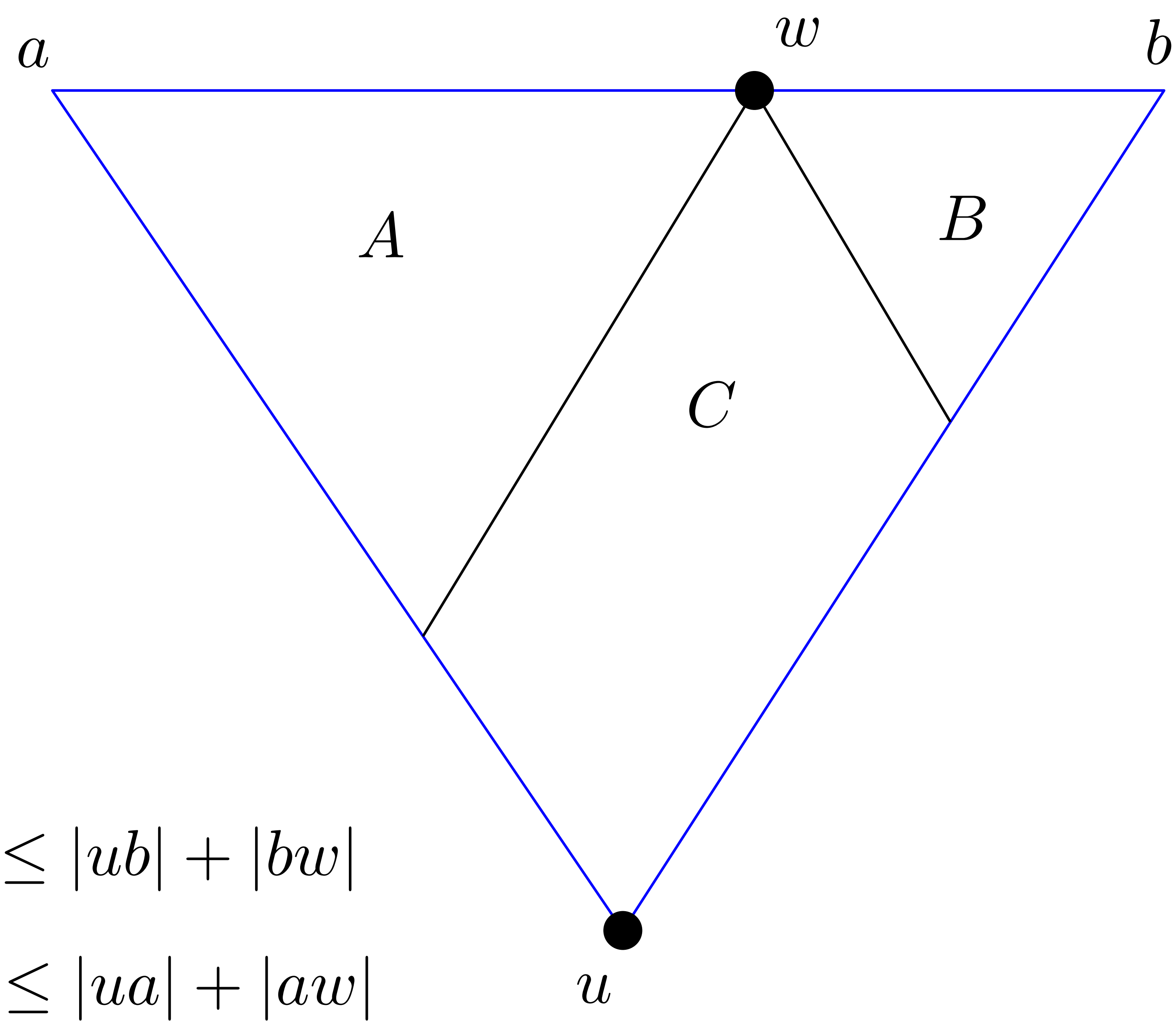

Induction hypothesis:

1. If A is empty, then $\delta(u, w) \leq |ub| + |bw|$
2. If B is empty, then $\delta(u, w) \leq |ua| + |aw|$
3. otherwise, $\delta(u, w) \leq \max\{|ua| + |aw|, |ub| + |bw|\}$

Connection between △-Delaunay and θ-graphs

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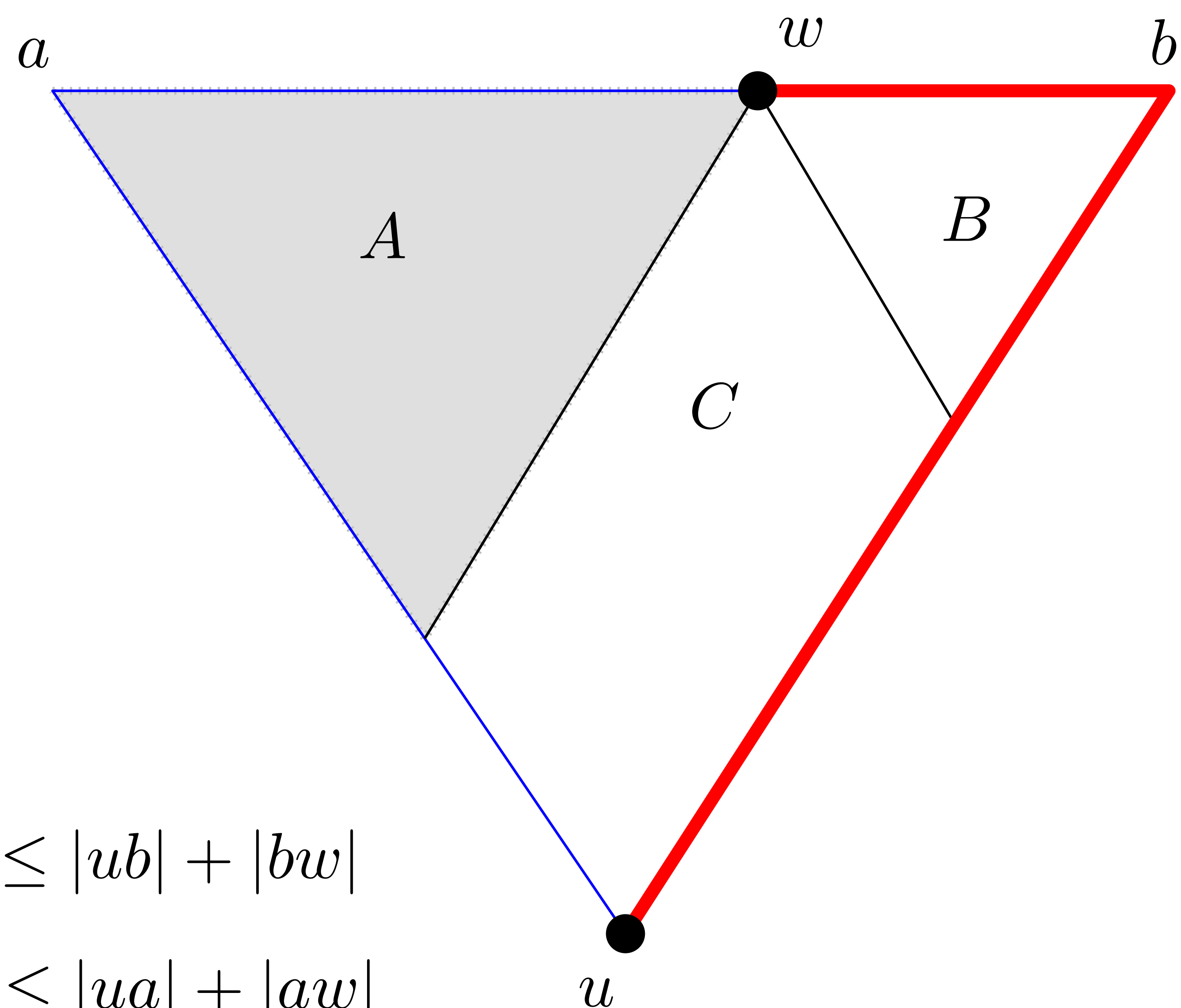

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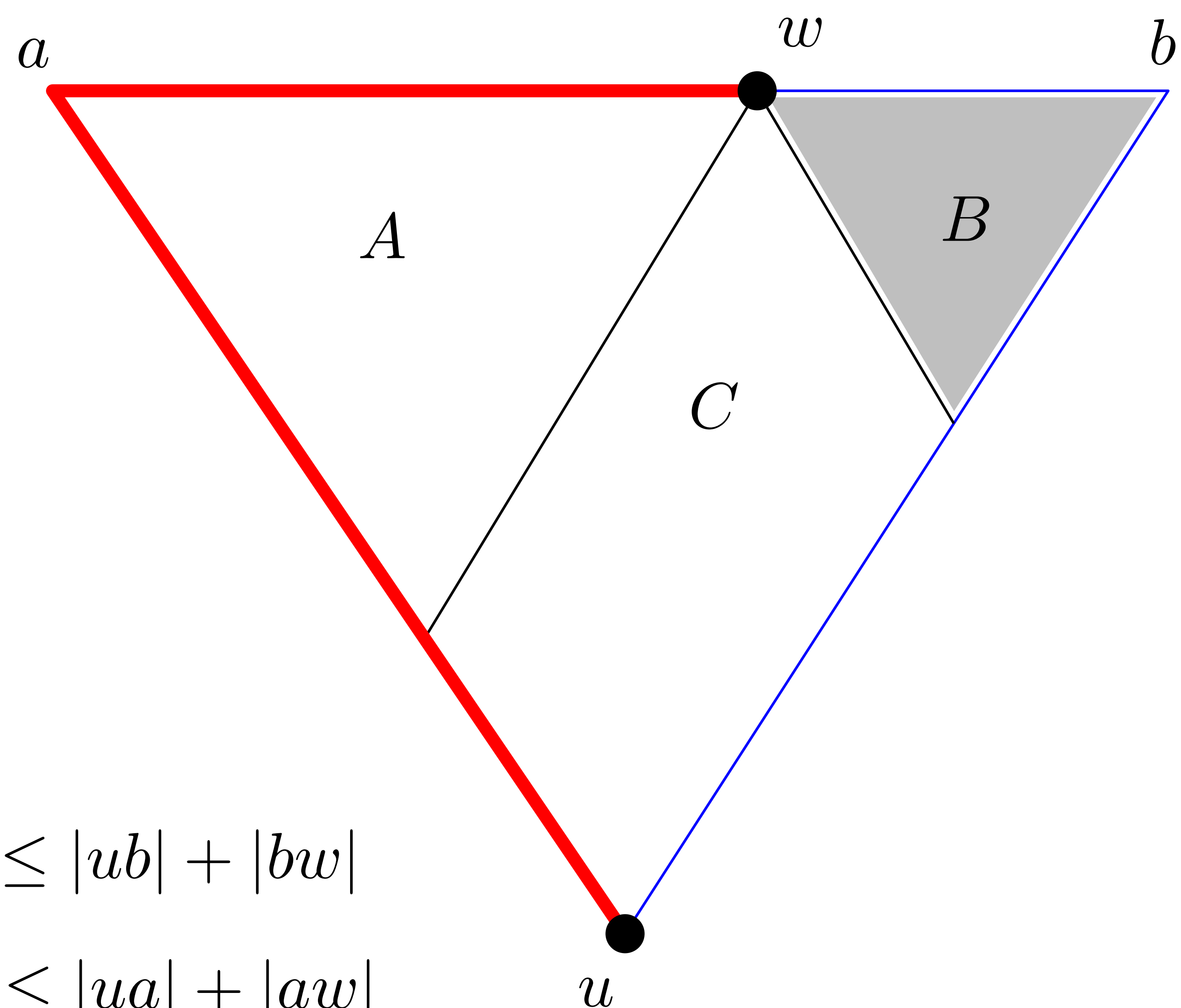

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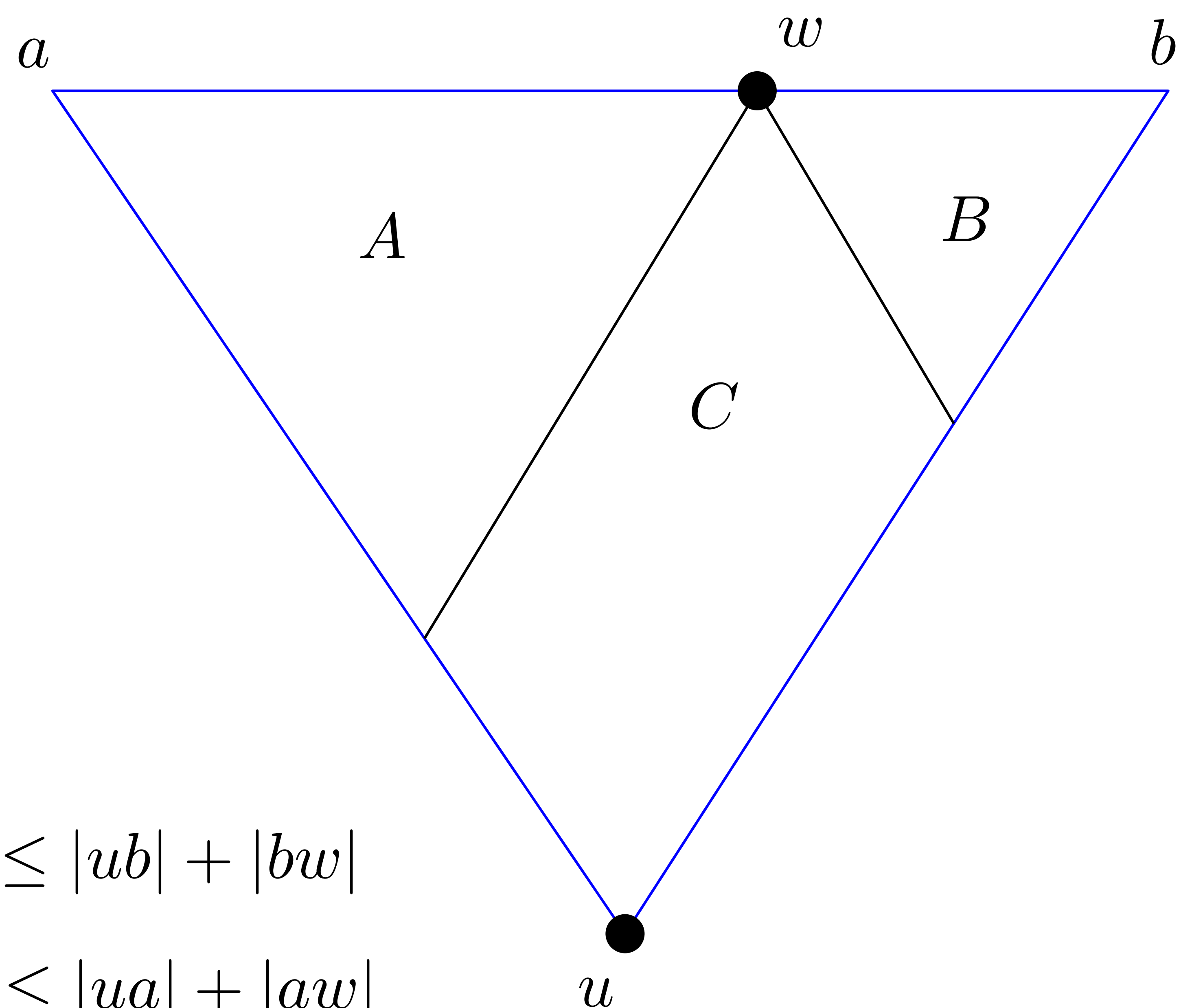

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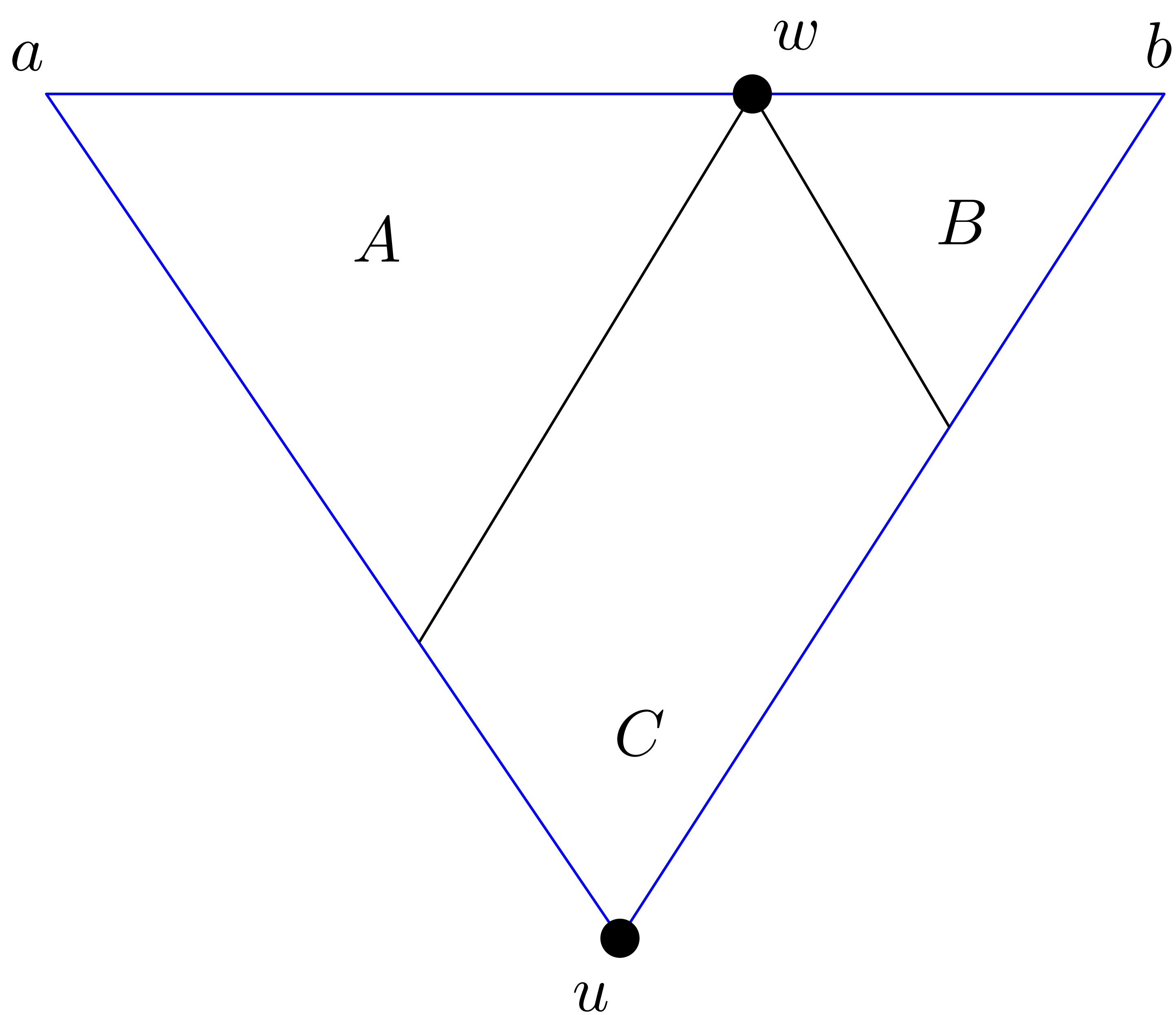

Generalized Delaunay Graphs:

B., Carmi, Collette, and Smid (2008) showed that the Delaunay graph defined by a distance function based on a convex shape C is a t-spanner where t depends only on the shape of the C.

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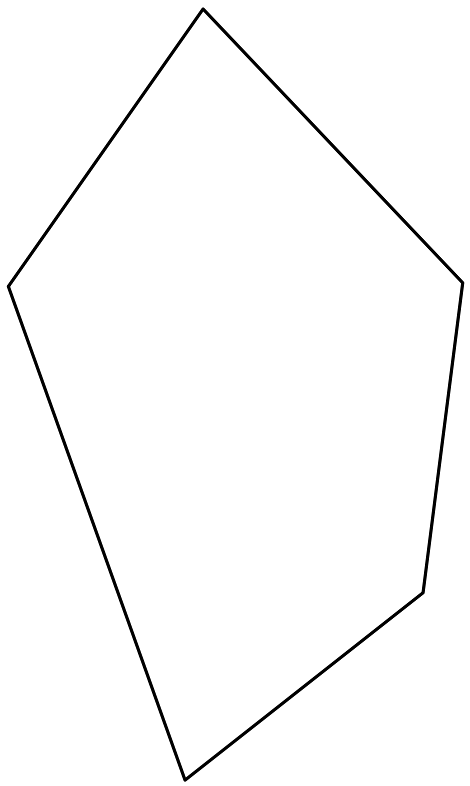

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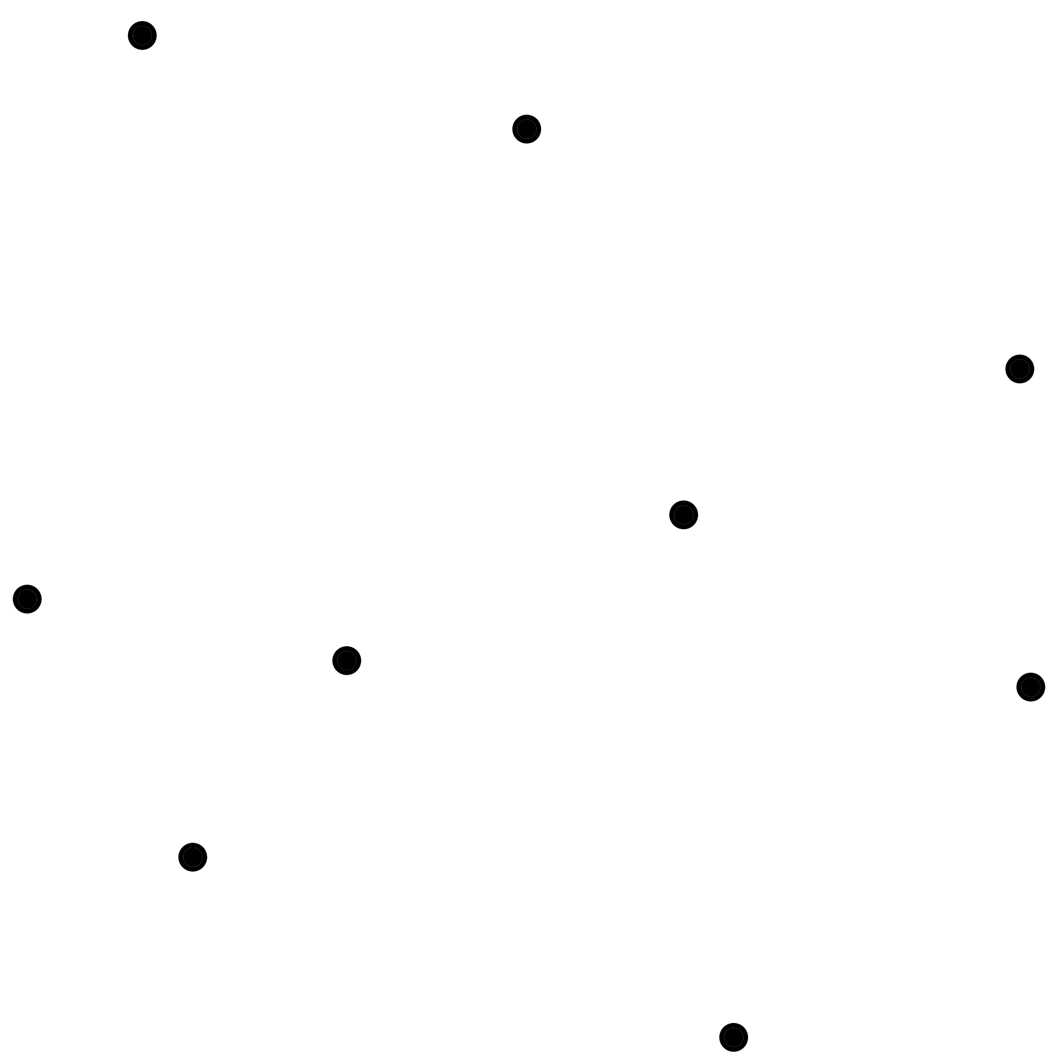

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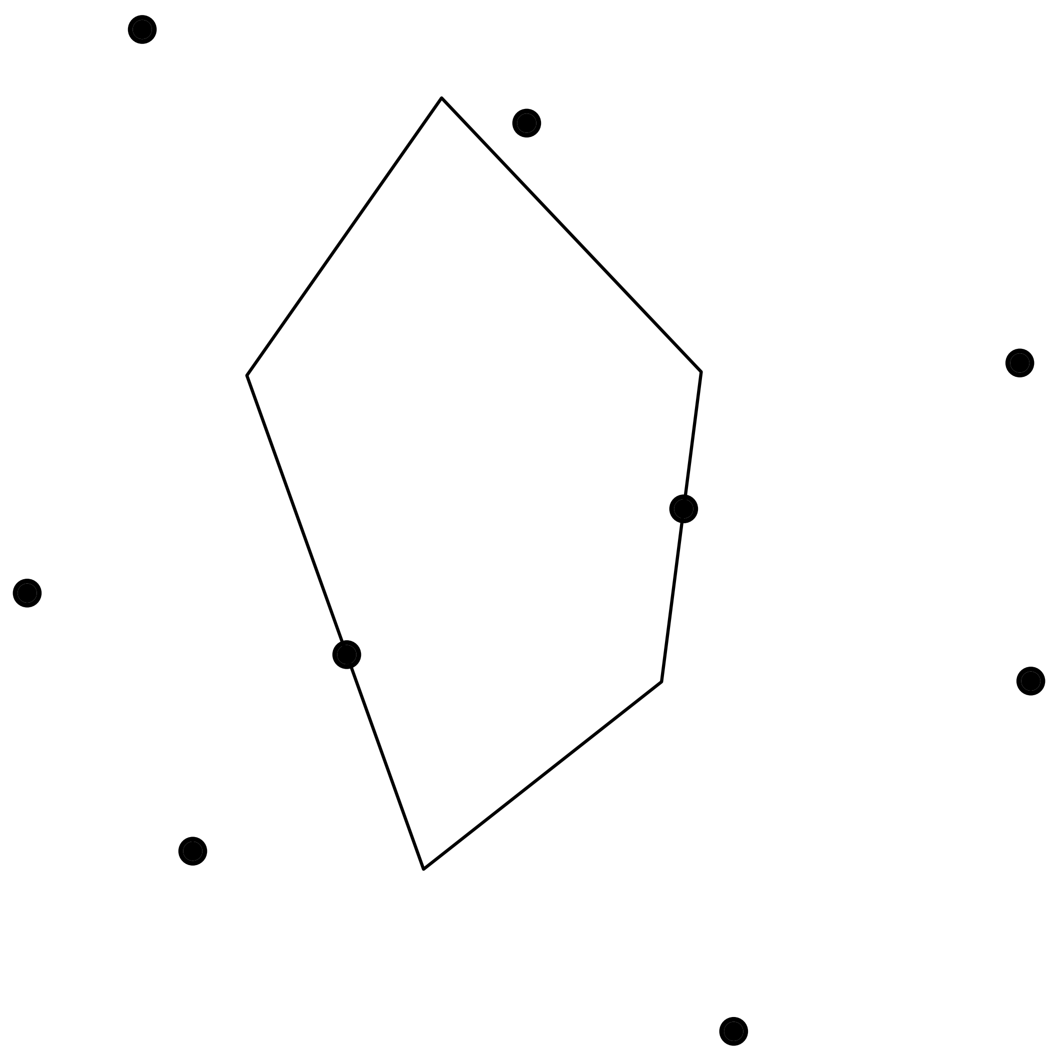

Spanning ratio depends on P(C)/W(C)

P(C) = Perimeter of C

W(C) = Width of C

Generalized Delaunay Graphs:

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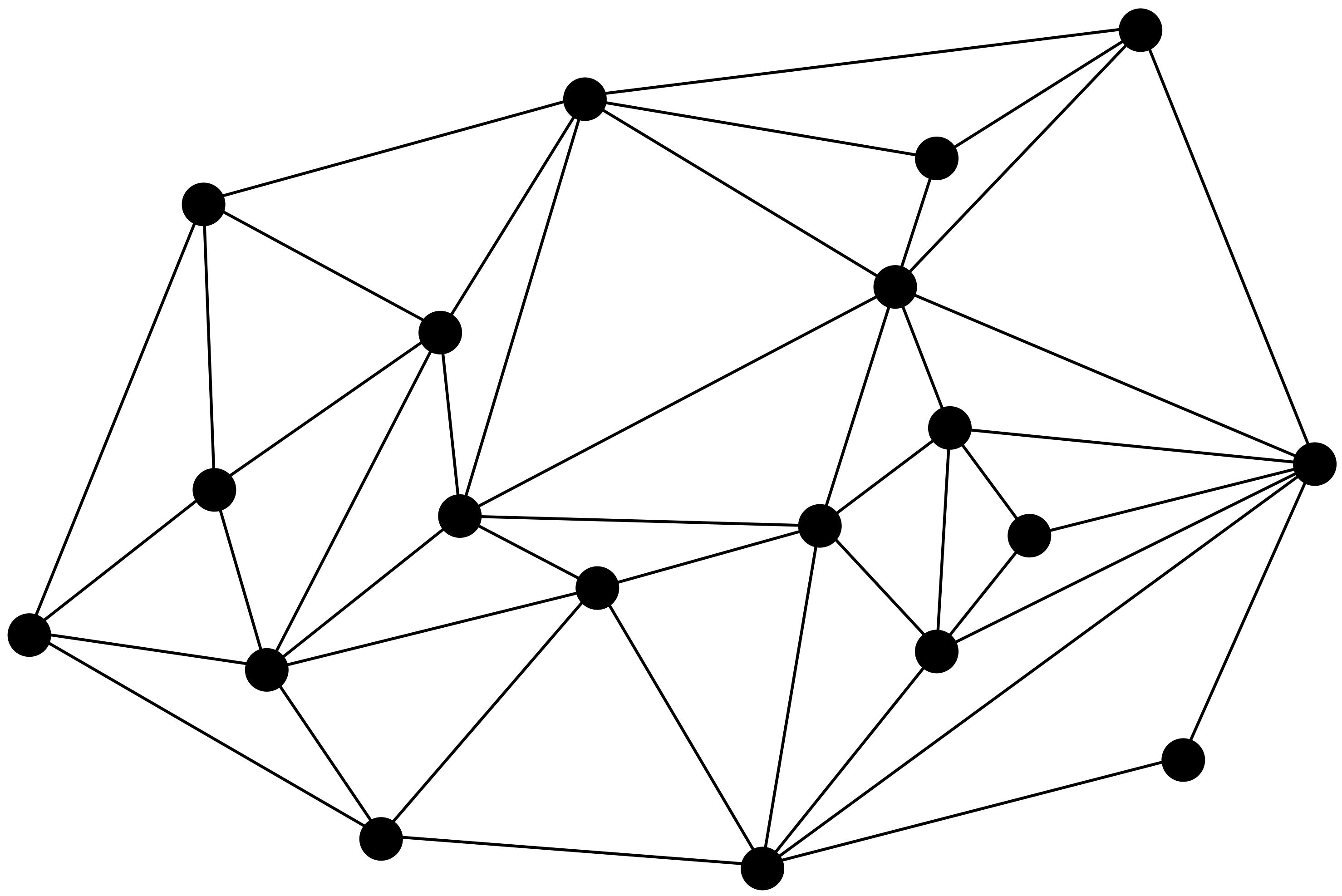
Main idea: Let G be the Delaunay graph based on convex shape C.

- G is plane.
- G satisfies the α-diamond property.
- The stretch factor of any one-sided path in G is bounded by a constant.
- Shortcuts can be made similar to Dobkin et al.'s proof when the path is not one-sided.

Diamond Property:

A planar graph has the α *diamond*-property provided that the following holds for every edge.



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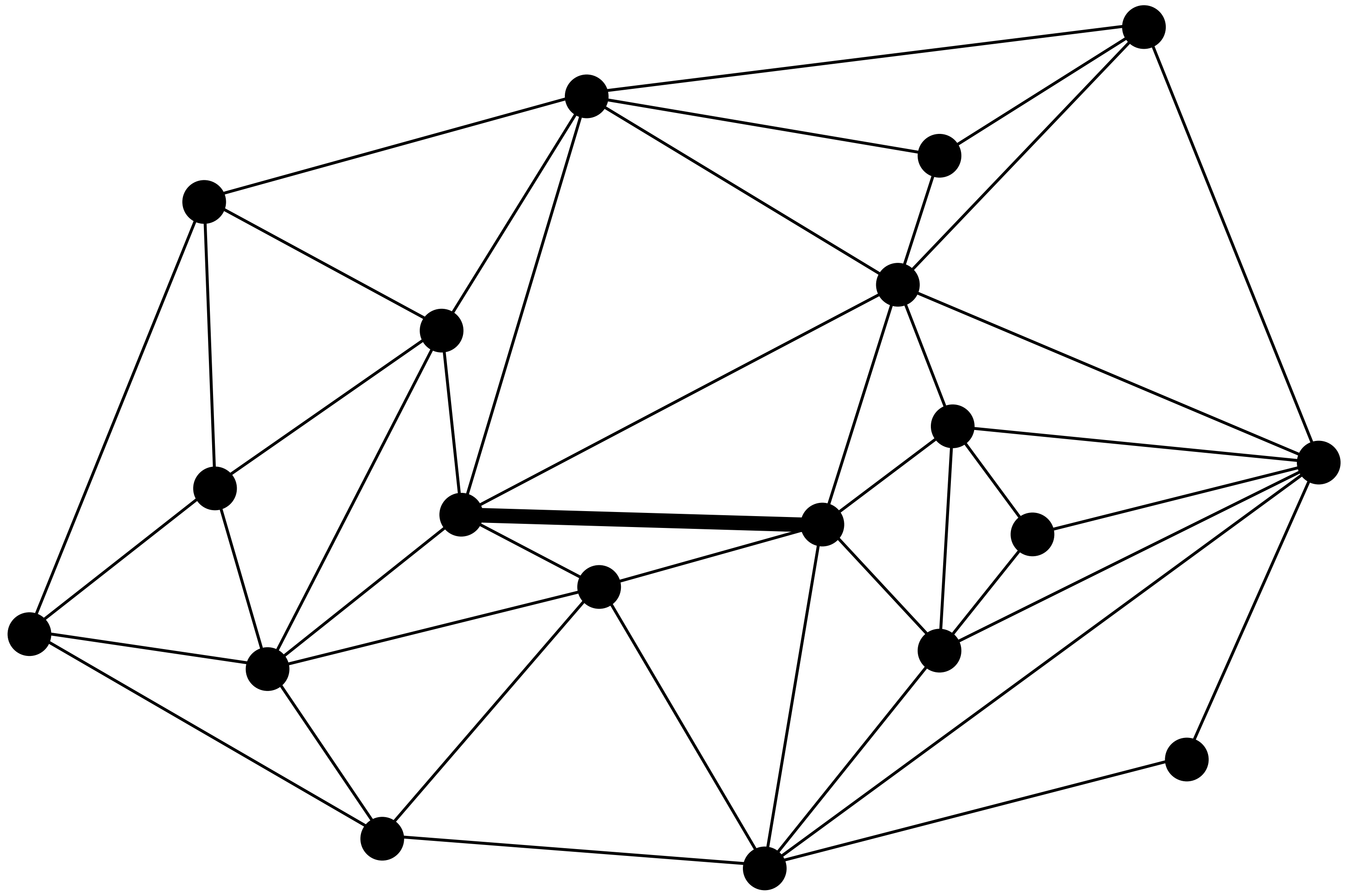

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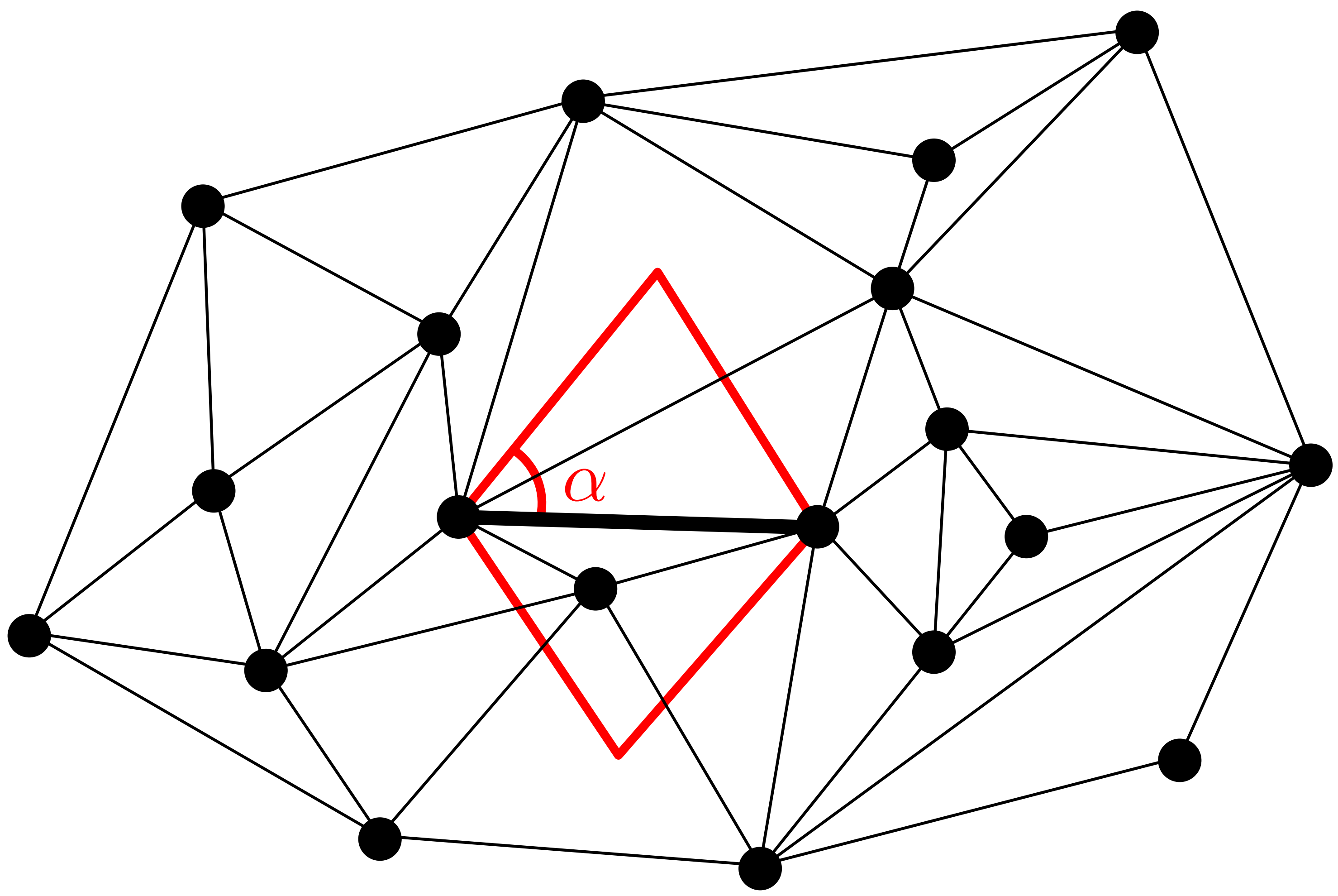

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- Das and Joseph (1989) showed that triangulations with the α-diamond property have spanning ratio $8\pi^2/\alpha^2 \sin^2(\alpha/4)$.
- B., Lee and Smid (2007) slightly improved the spanning ratio $8(\pi-\alpha)^2/\alpha^2 \sin^2(\alpha/4)$.

The Delaunay triangulation, the greedy triangulation and the minimum weight triangulation are all α-diamond triangulations for some constant α.

Question:

Are there bounded degree subgraphs of Delaunay graphs that are constant spanners?

Bounded Degree Spanners:

Authors	Deg	Ratio
B., Gudmundsson, and Smid (2002)	27	8.27
Li and Wang (2003)	23	6.44
B., Smid and Xu (2006)	17	23.58
Kanj and Perkovic (2008)	14	2.92
B., Hill and Smid (2018)	8	4.42
B., Carmi and Chaitman (2010)	7	11.6
Bonichon et al. (2010)	6	6
Bonichon et al. (2015)	4	156.82
Kanj et al. (2017)	4	20

Bounded Degree - Proof Idea:

B., Smid, Xu (2006)

Compute a Delaunay Triangulation of the point set.

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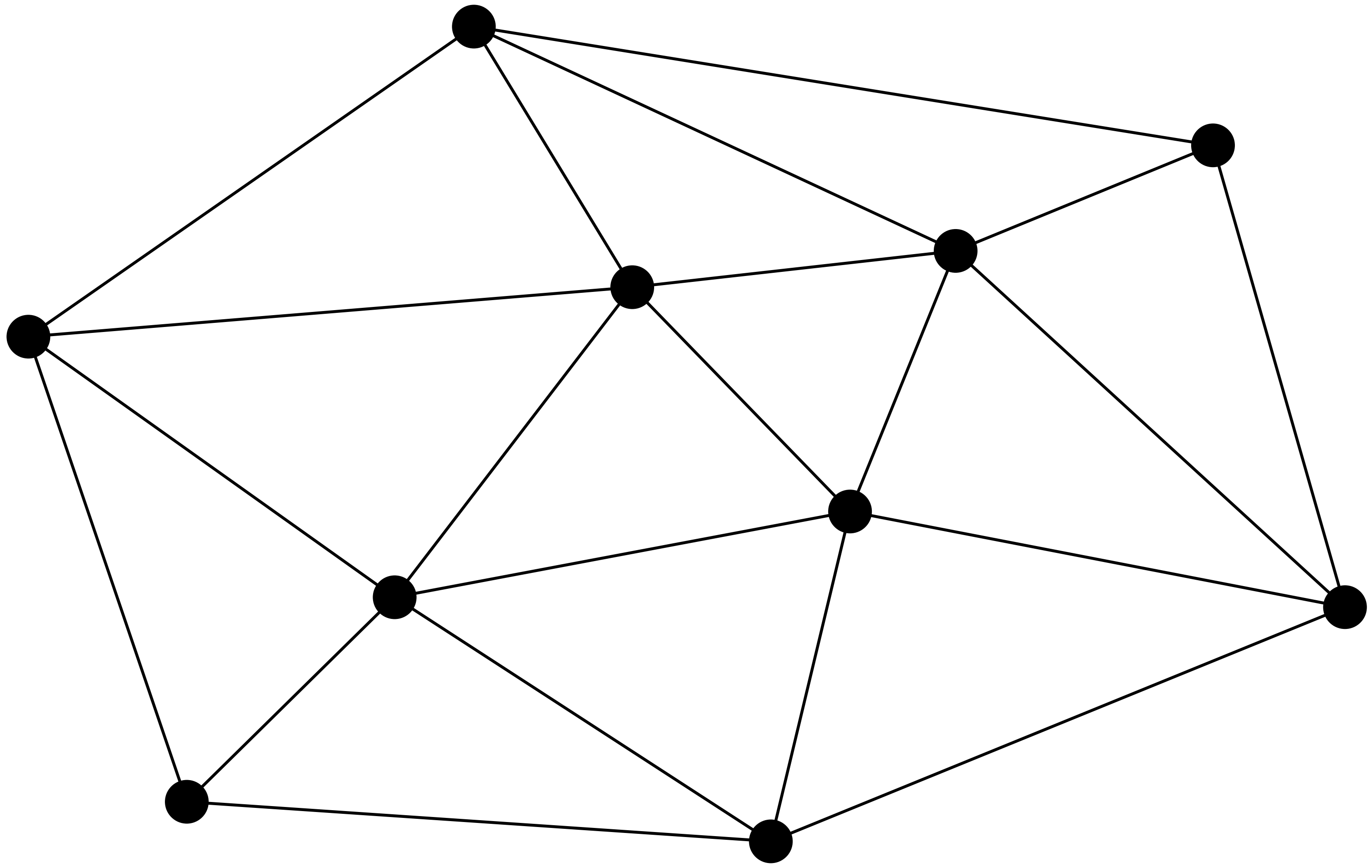

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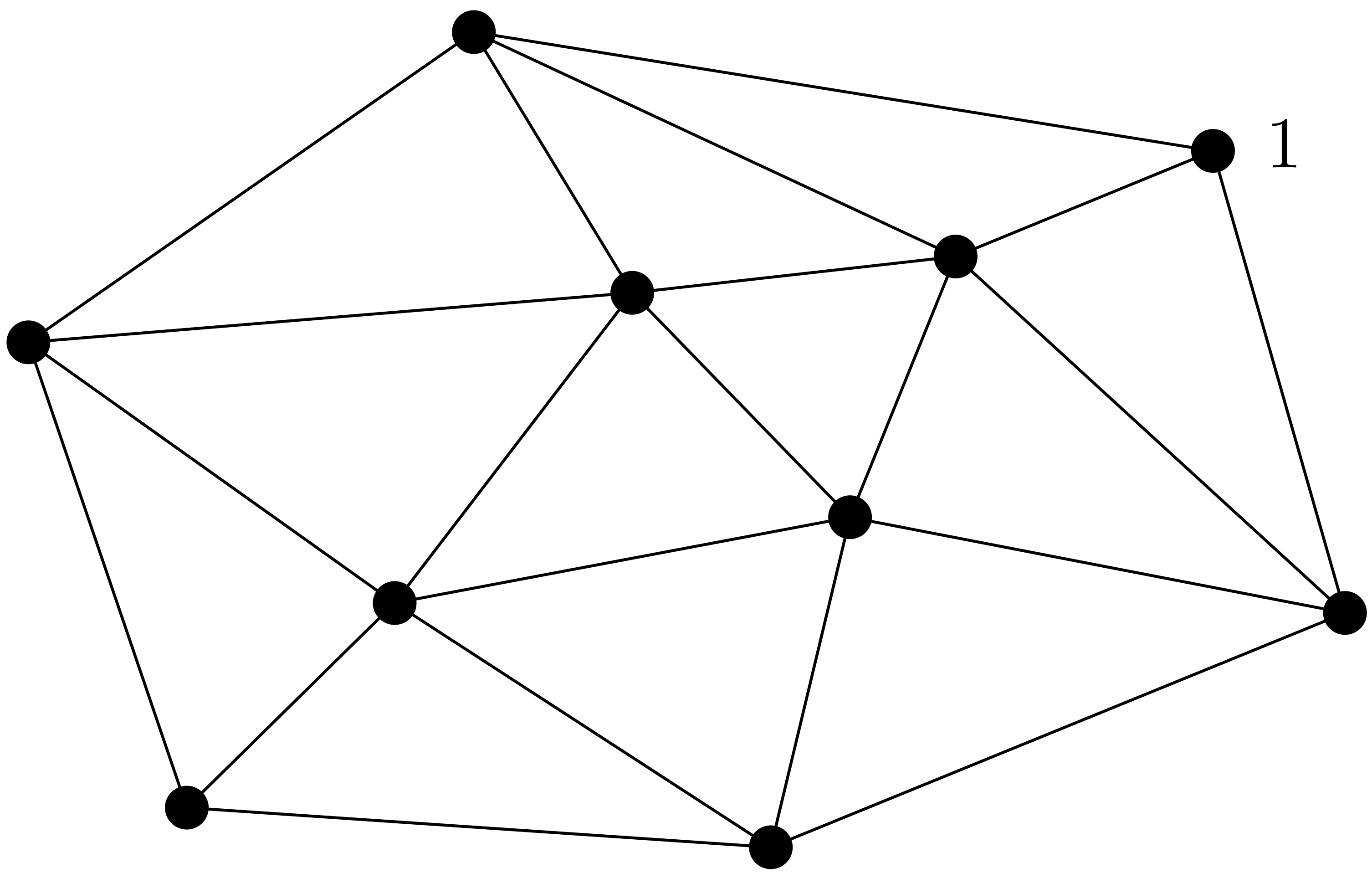

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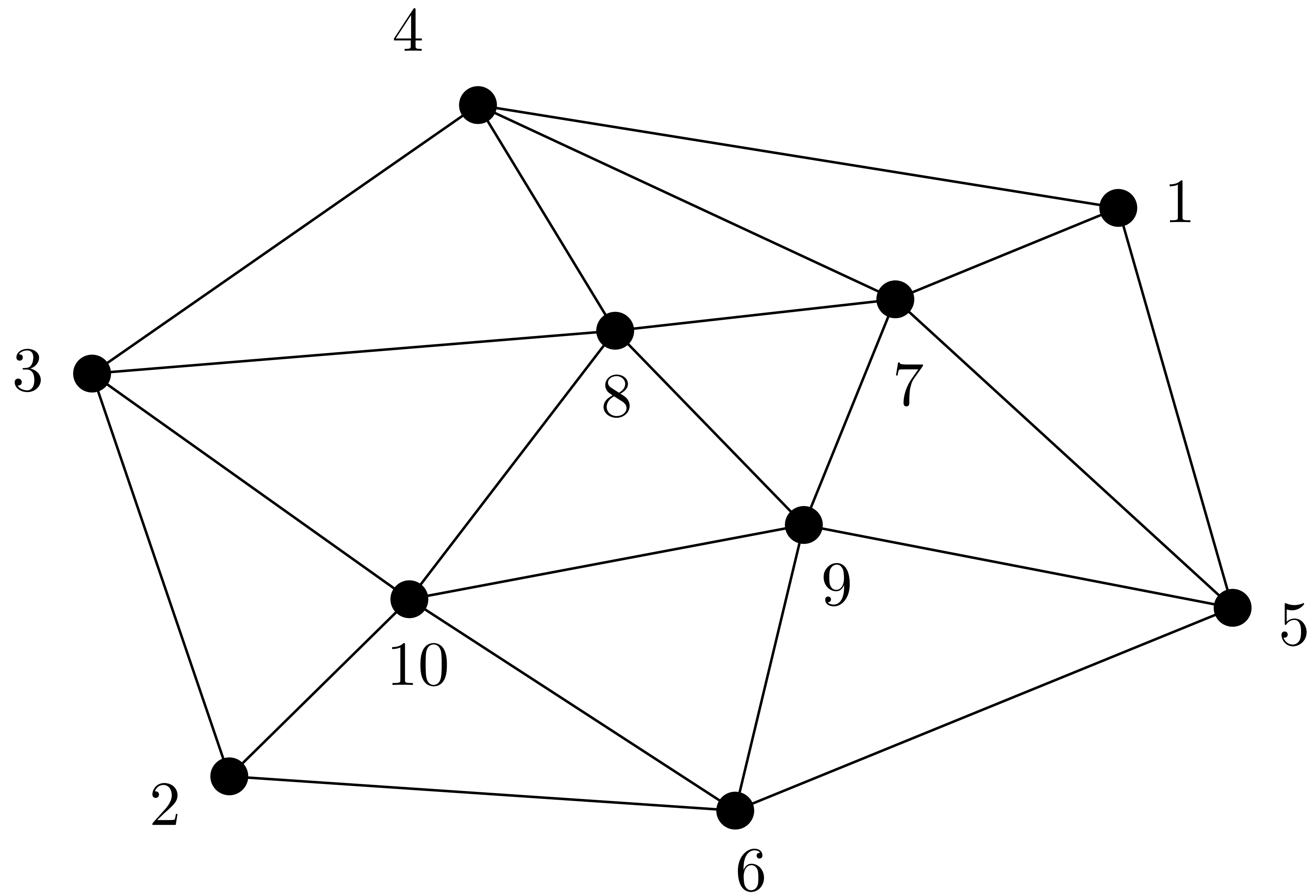

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Build the spanner by processing each vertex in reverse order.

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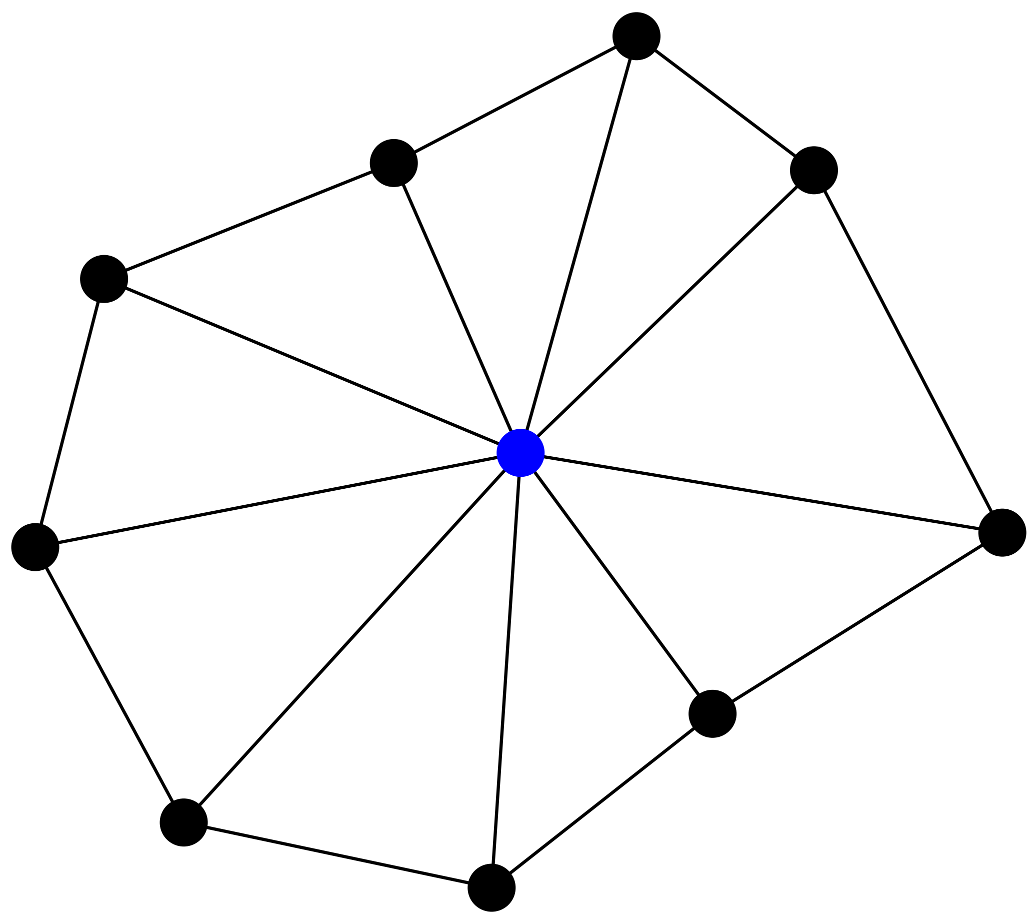

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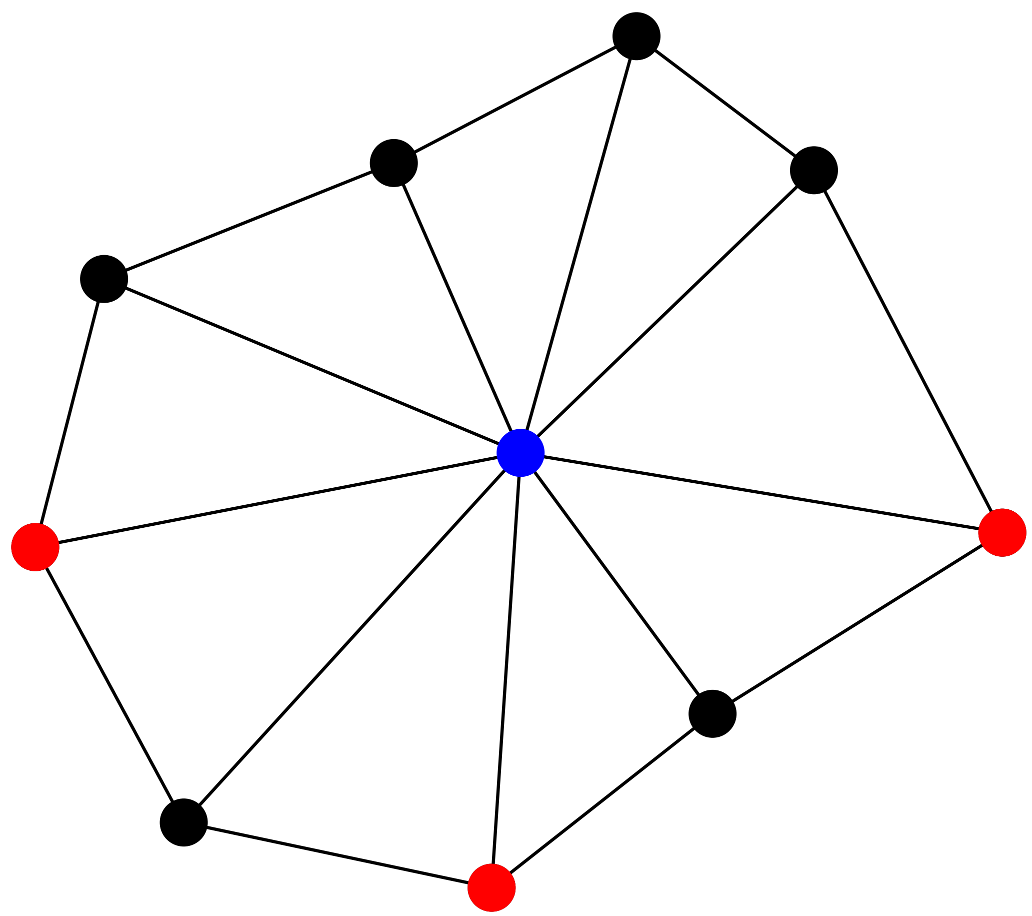

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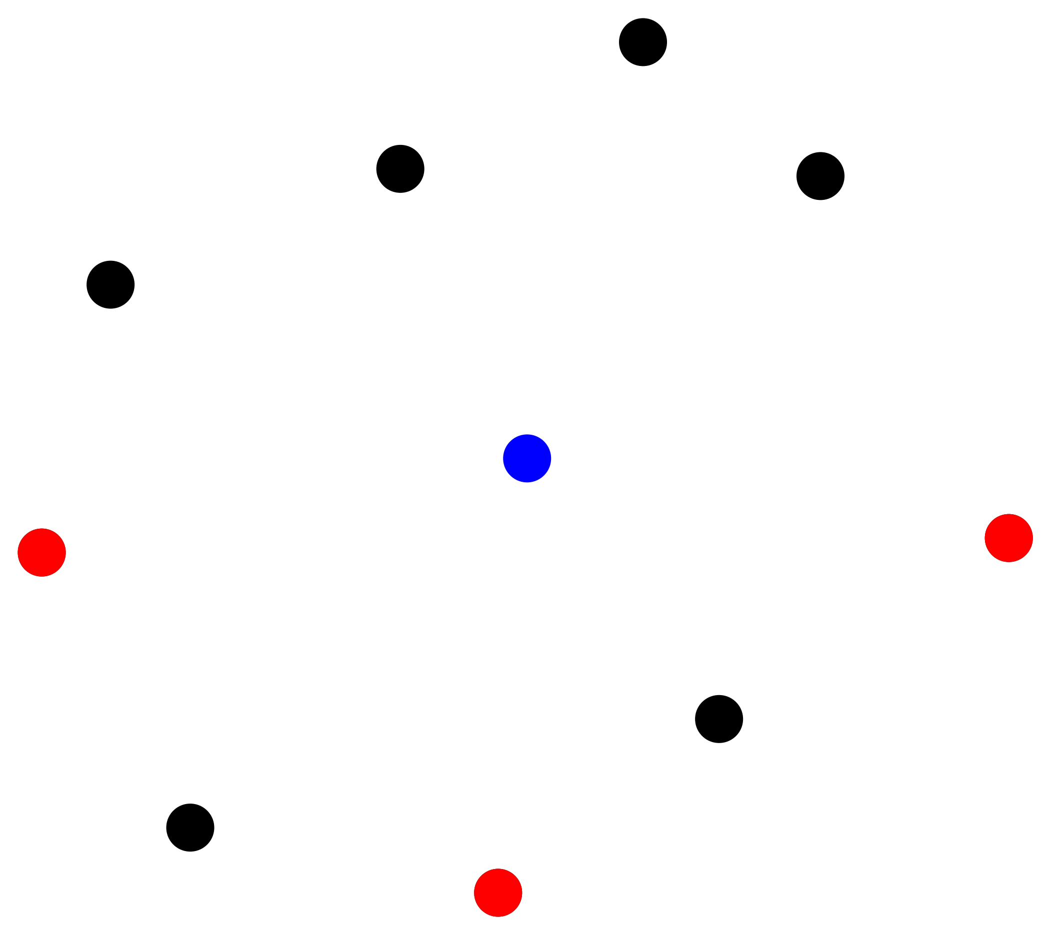

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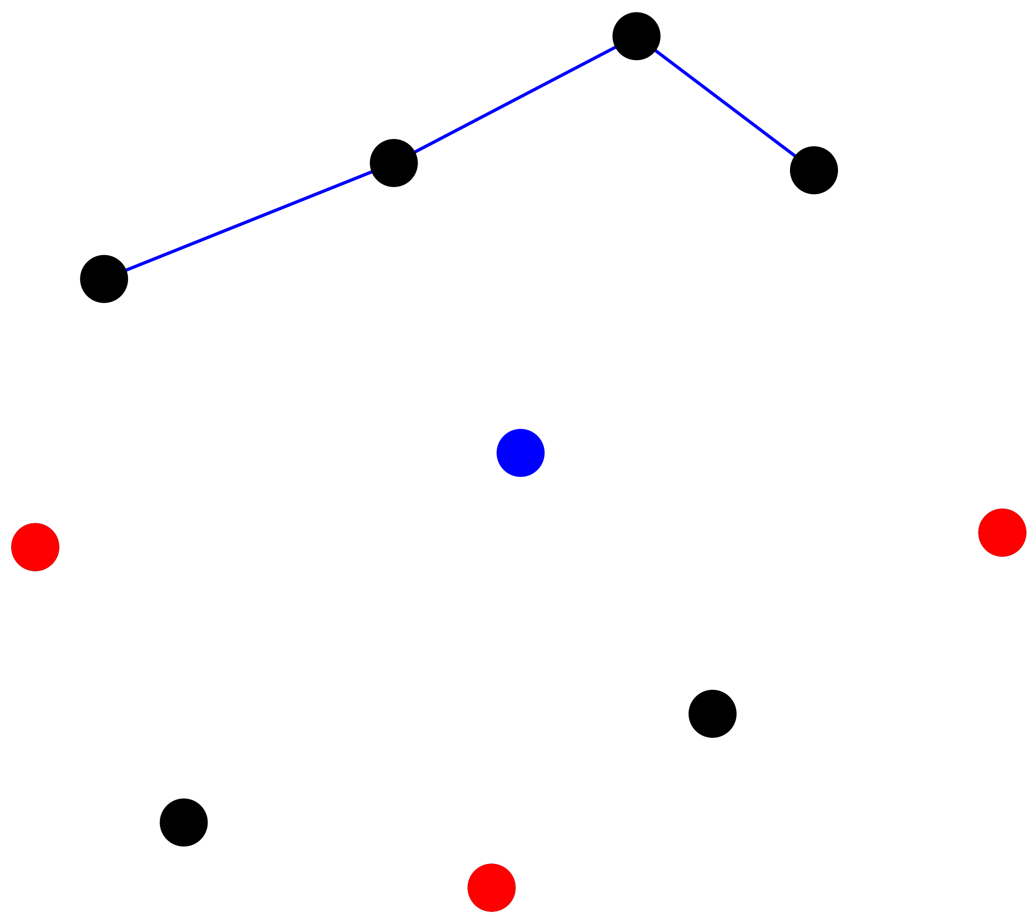

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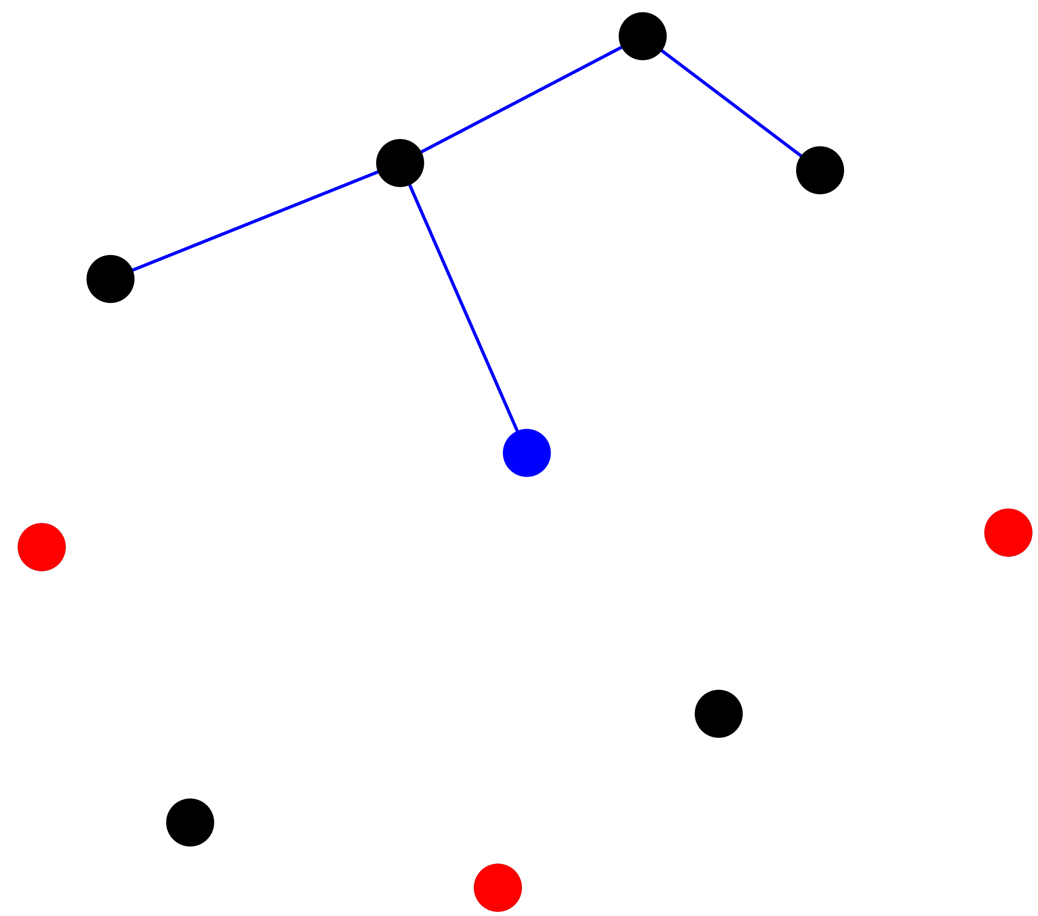

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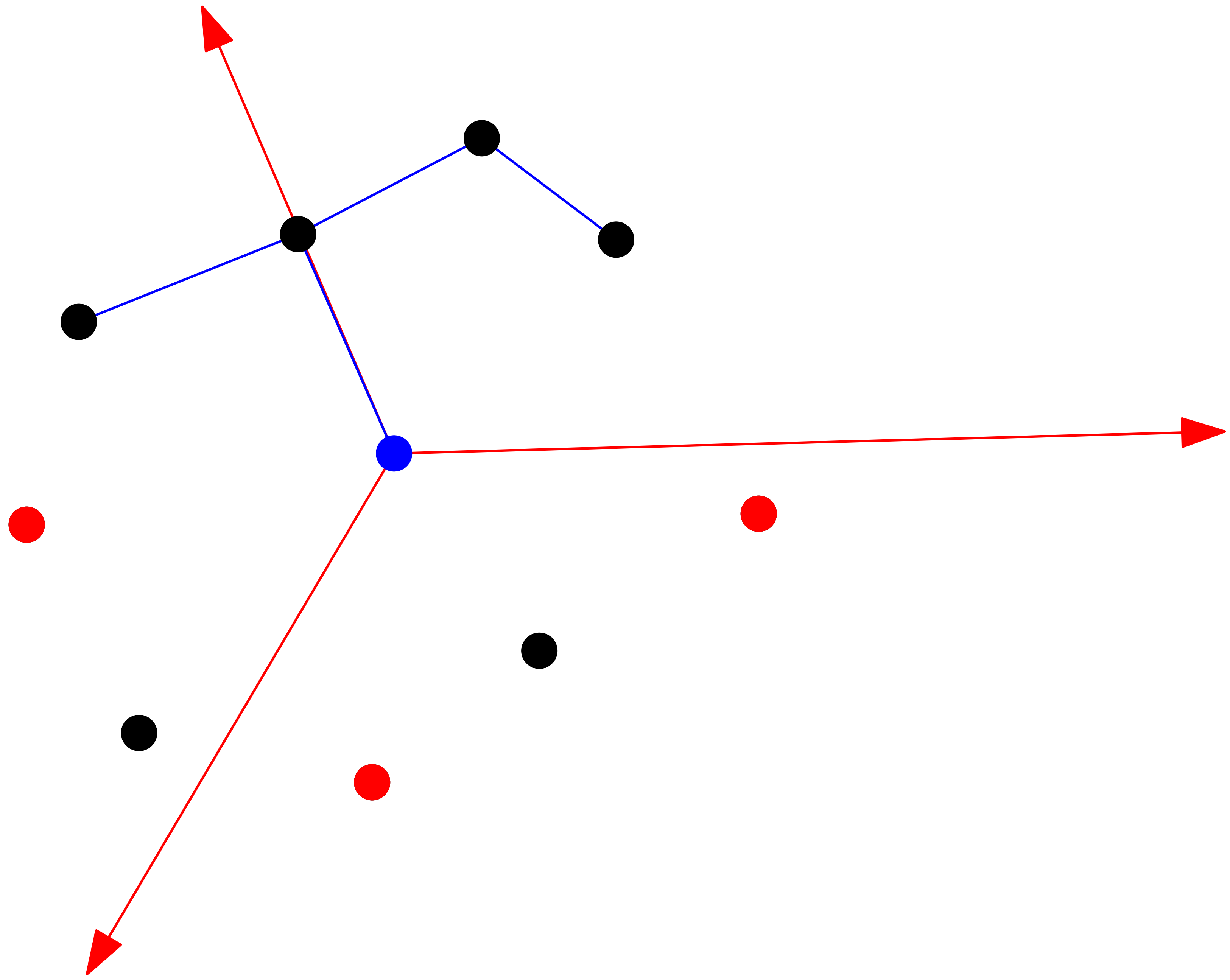

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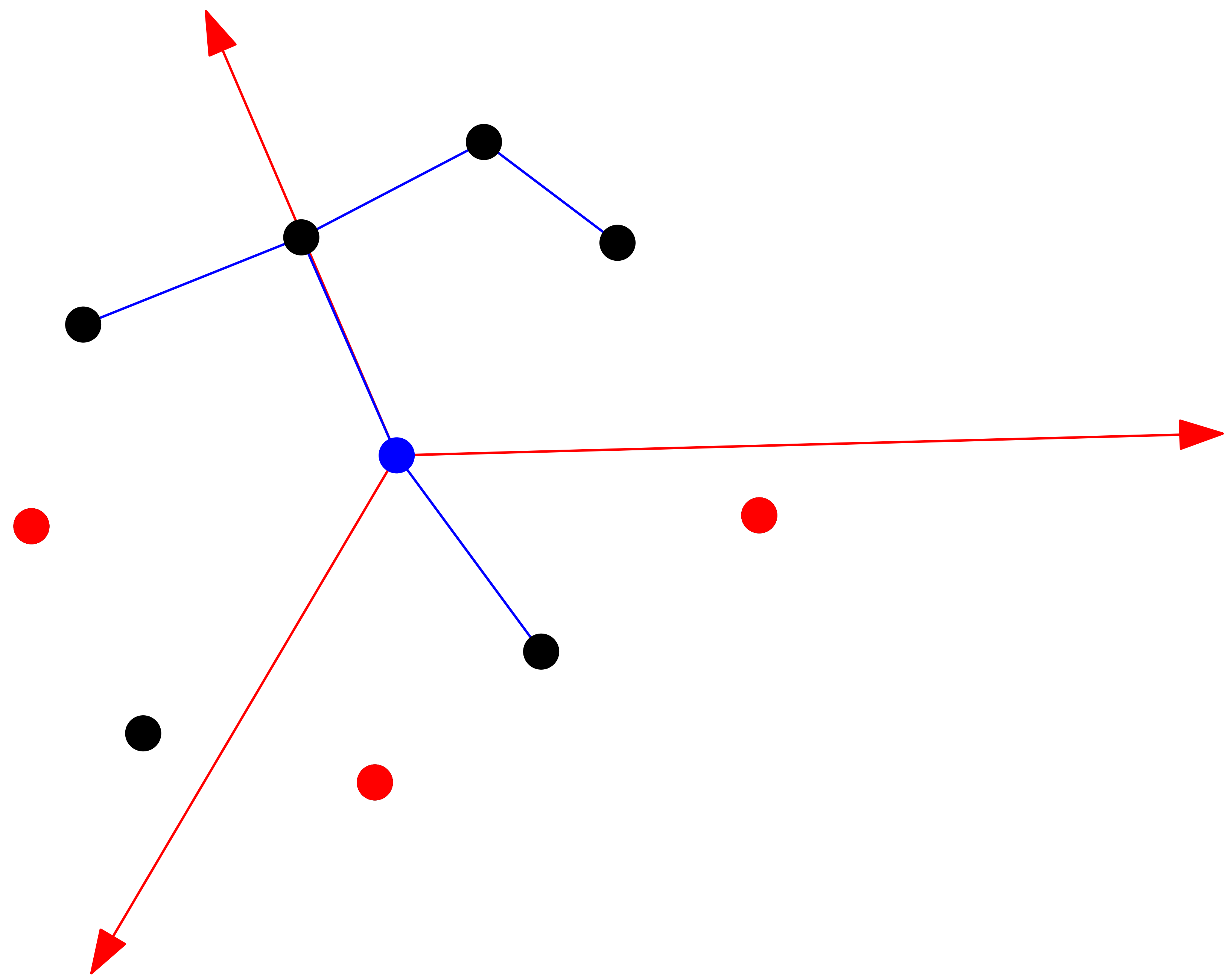

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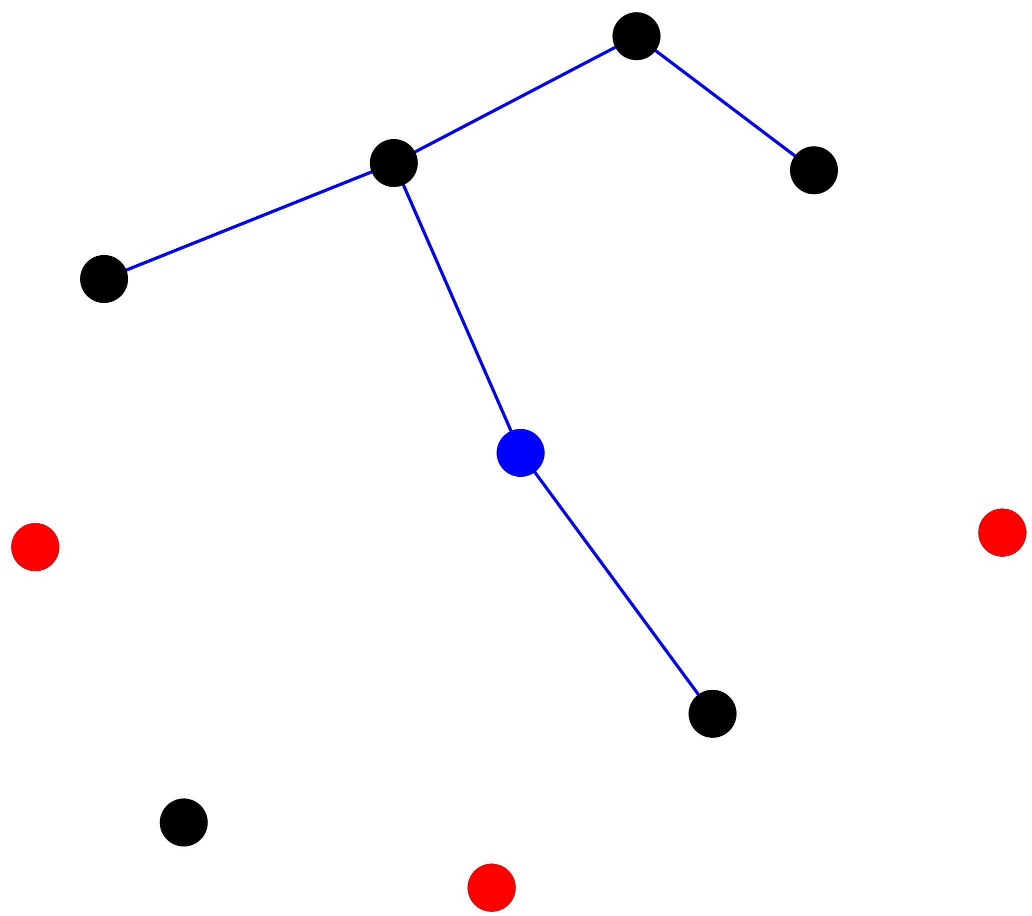
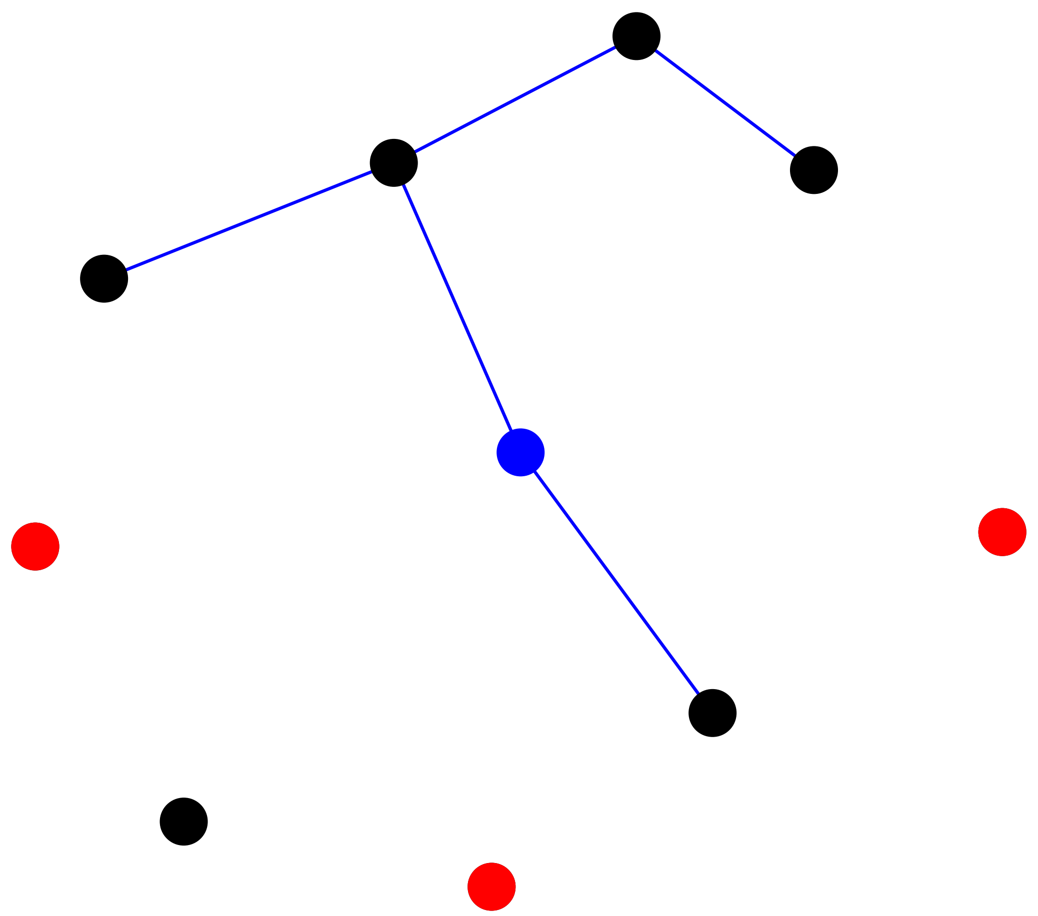


Bounded Degree - Proof Idea:

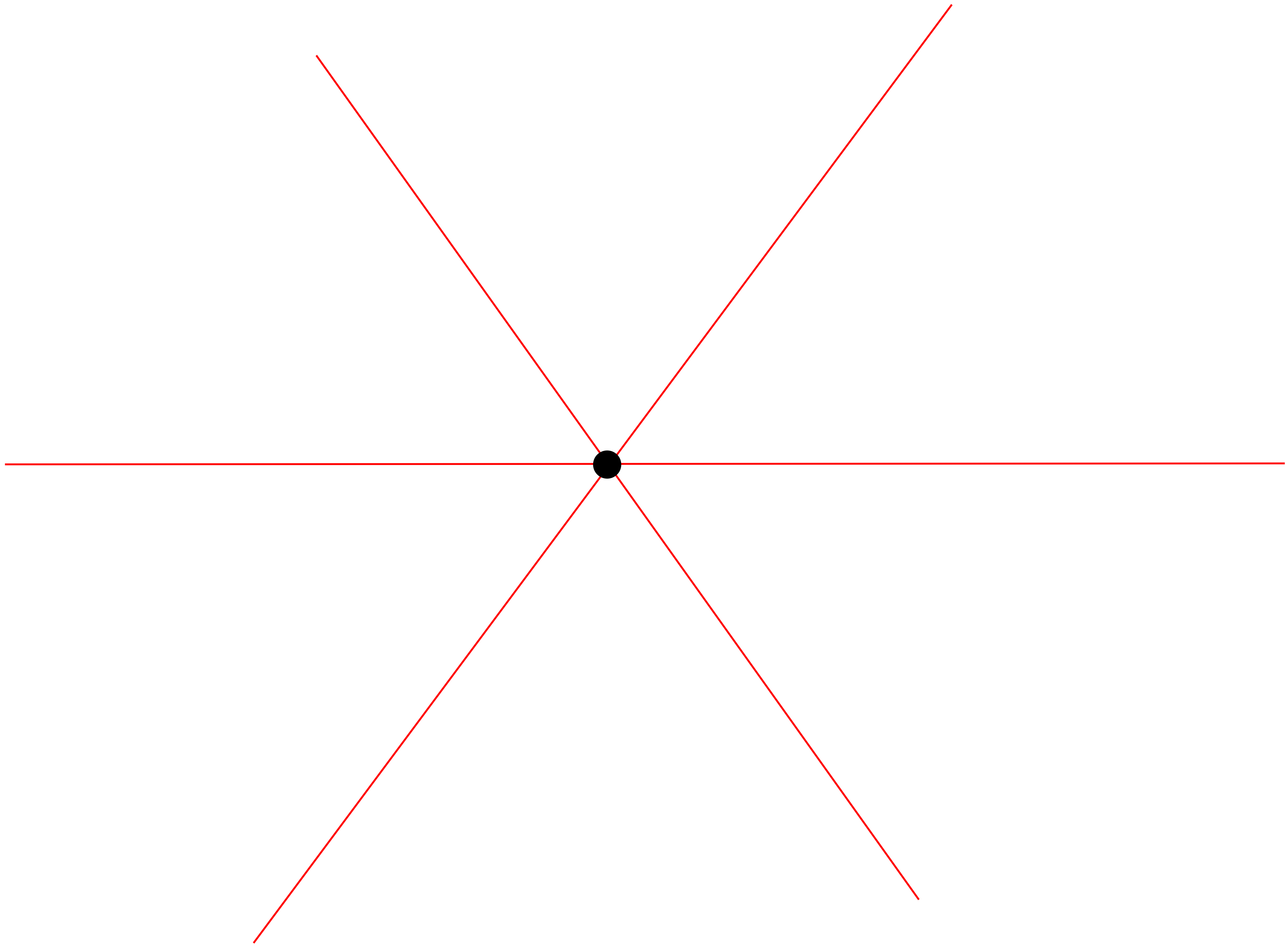
After a vertex is processed, never touch it again.

At the end of the construction, every vertex has degree at most 17.



Degree six 6-spanner:

Bonichon, Gavoille, Hanusse, Perkovic (2010)



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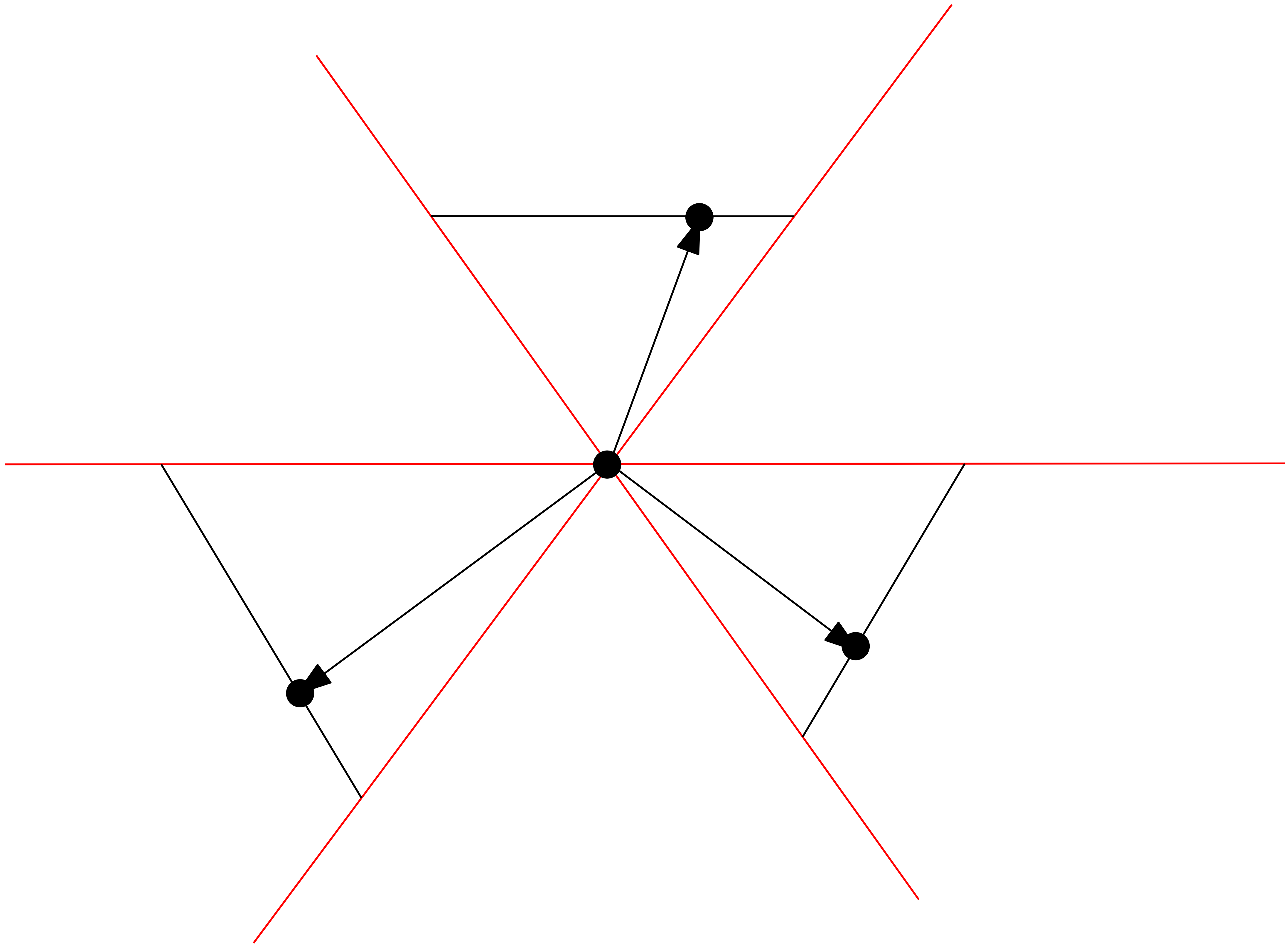

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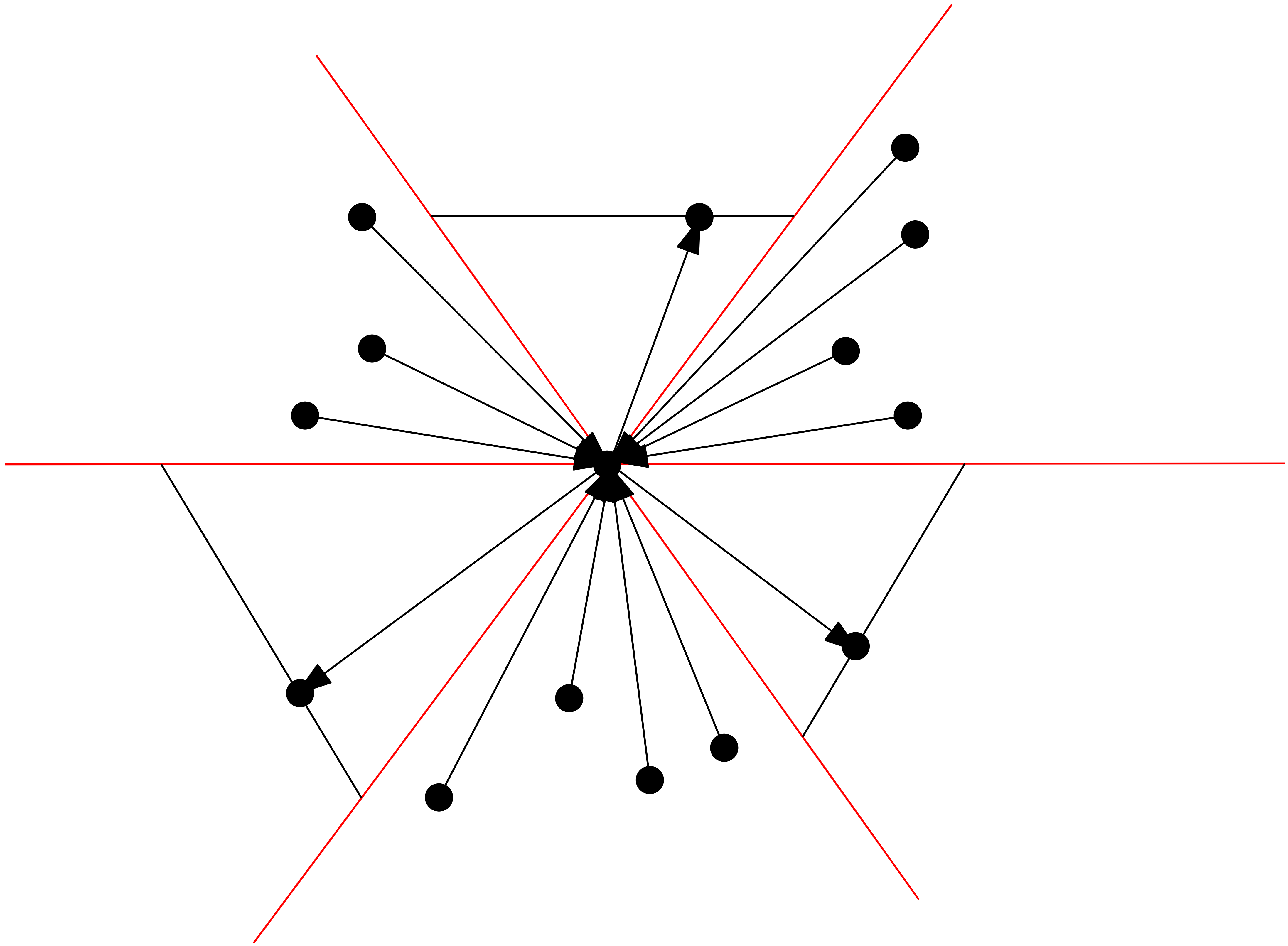

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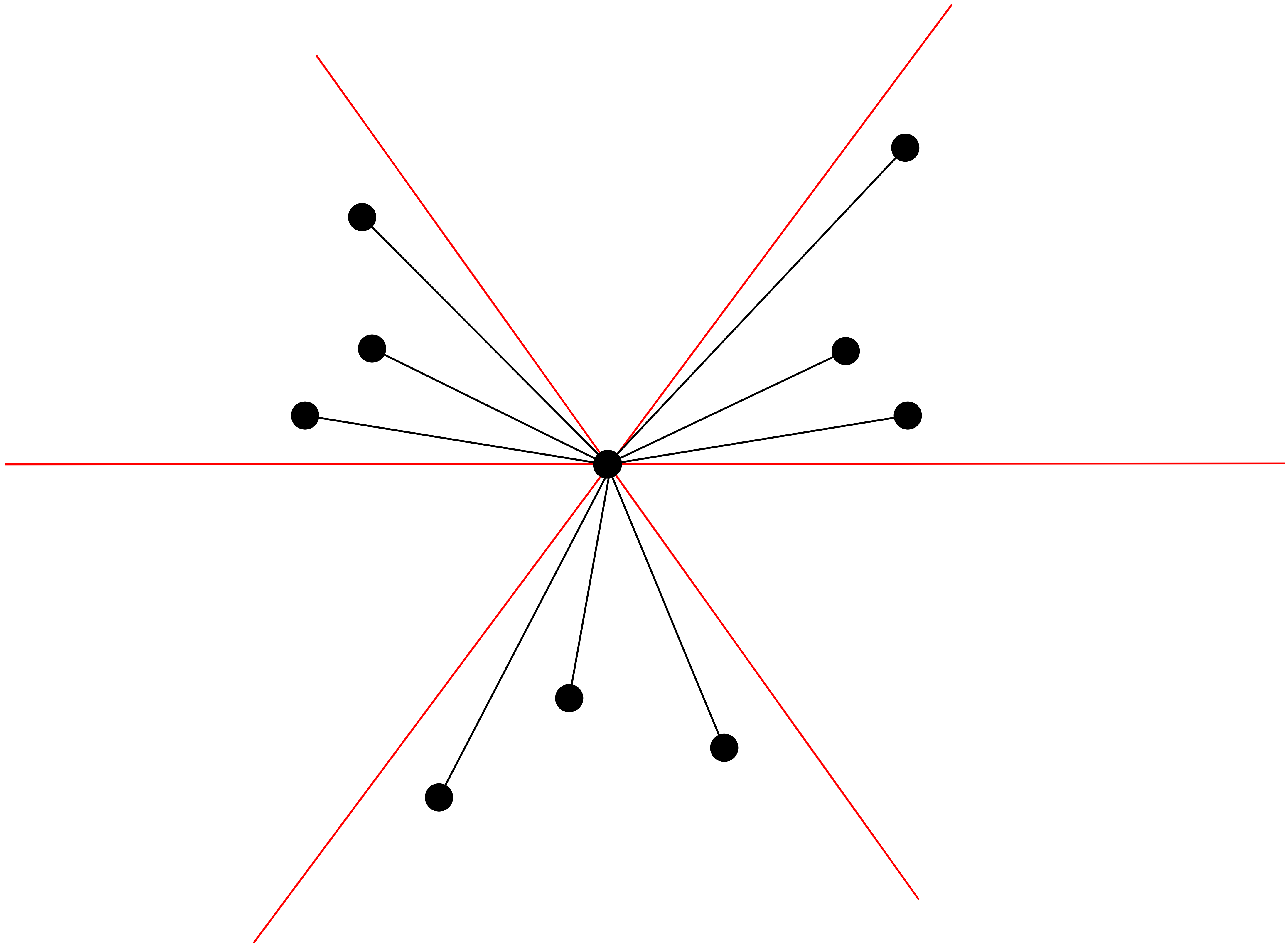

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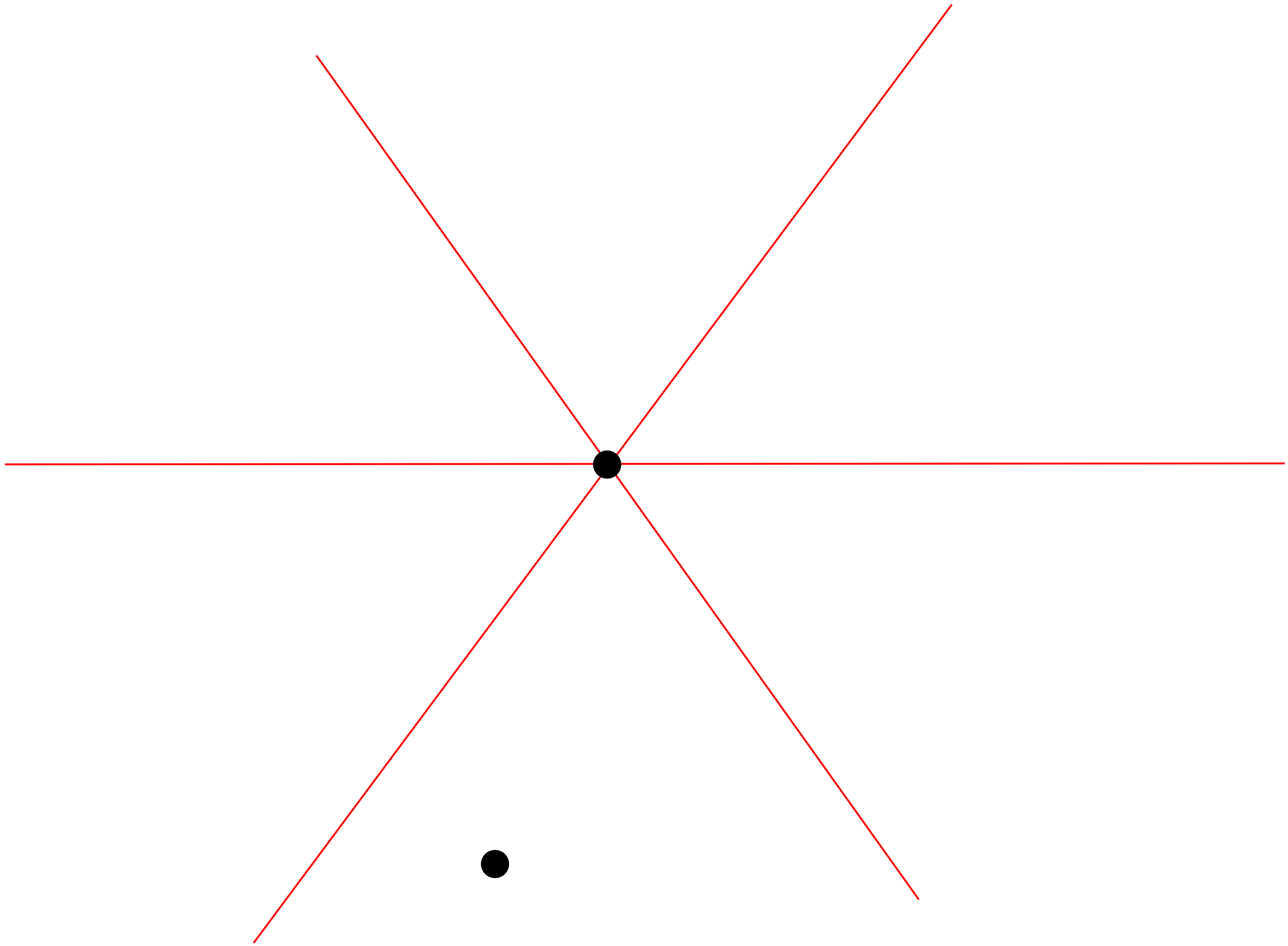

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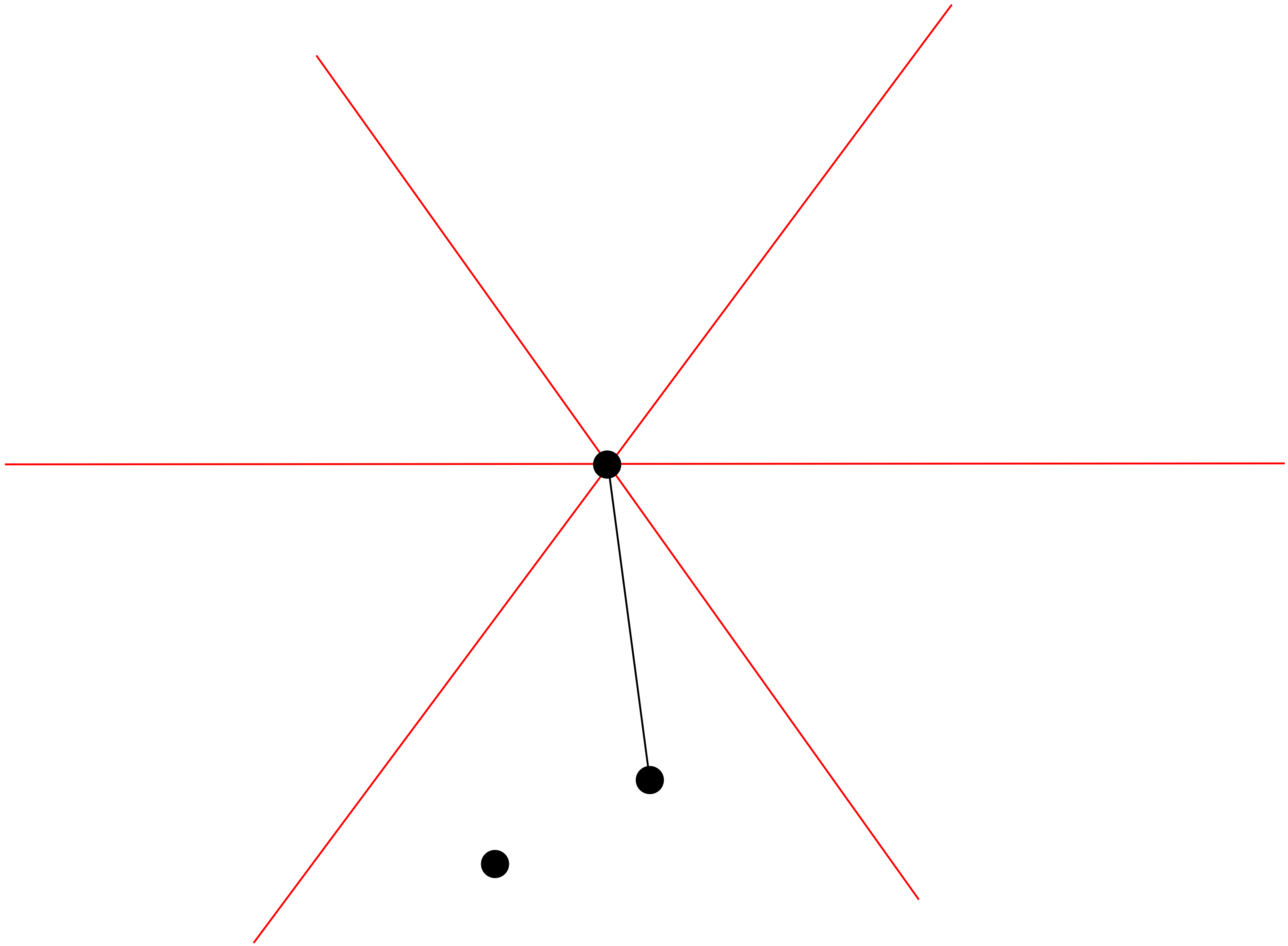

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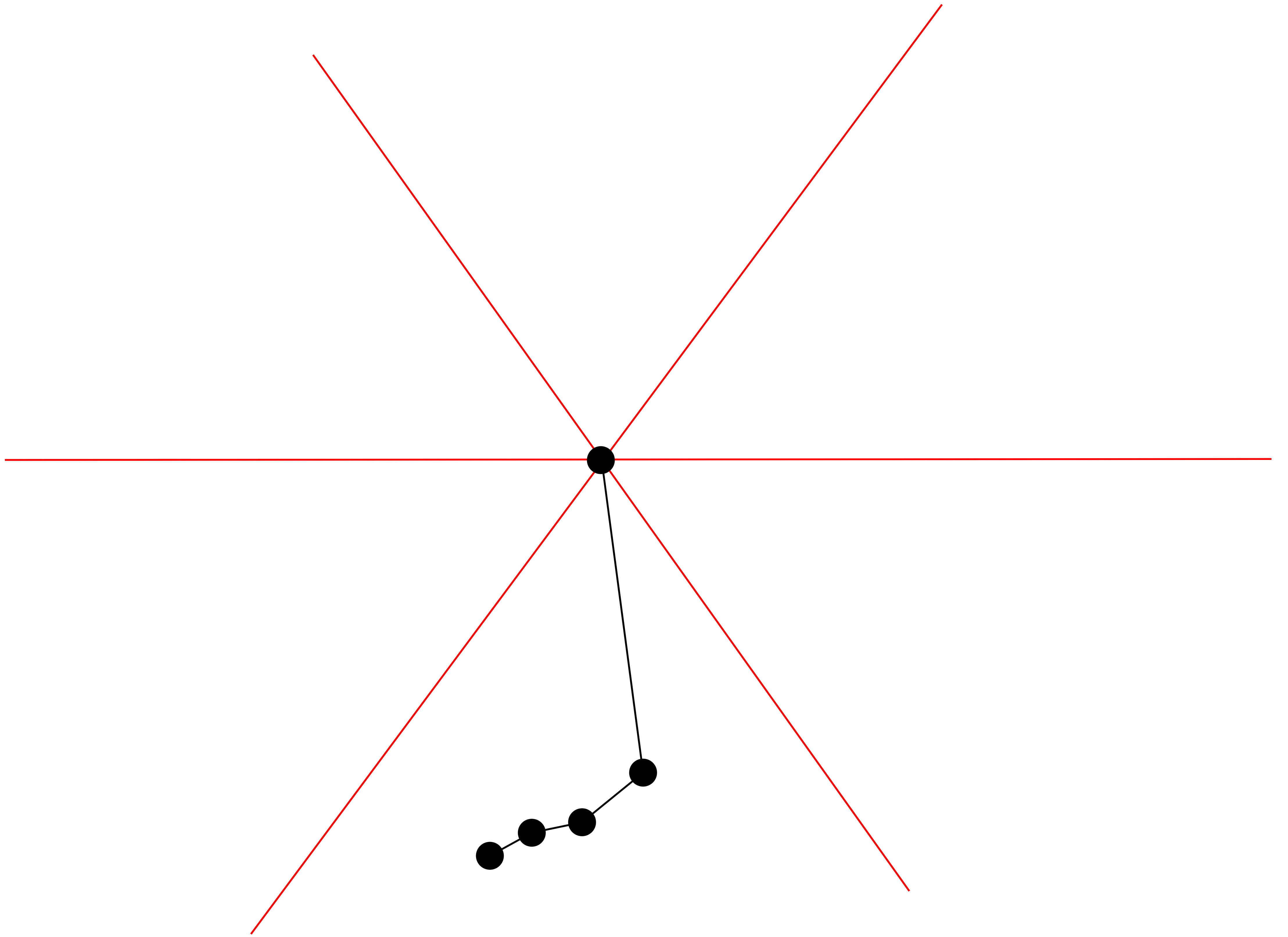

Open Question:

What is the smallest degree that can be achieved?

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- If planarity constraint is removed, Das and Heffernan (1996) showed that one can construct graphs with constant spanning ratio and maximum degree 3.
- With planarity constraint, is there a lower bound that is greater than 3? That is can we show the following:
  $\forall t\ \exists n\ \exists |P| = n$ every max degree 3 plane graph with vertex set P has spanning ratio greater than t.

Question:

Can we do better if points are in convex position?

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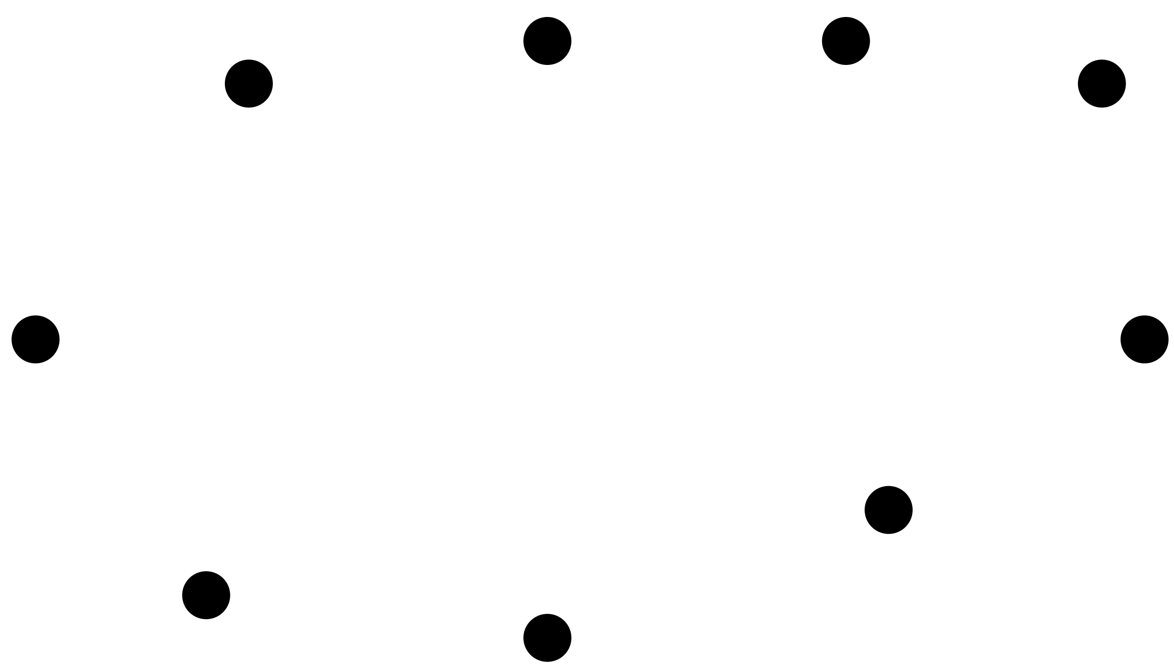
Authors	Deg	Ratio
Kanj et al. (2017)	3	20
Biniaz et al. (2017)	3	5.19
Bakhshesh and Farshi (2020)	3	5.19

Question:

Can we do better if points are in convex position?

Algorithm of Biniaz et al. (2017):



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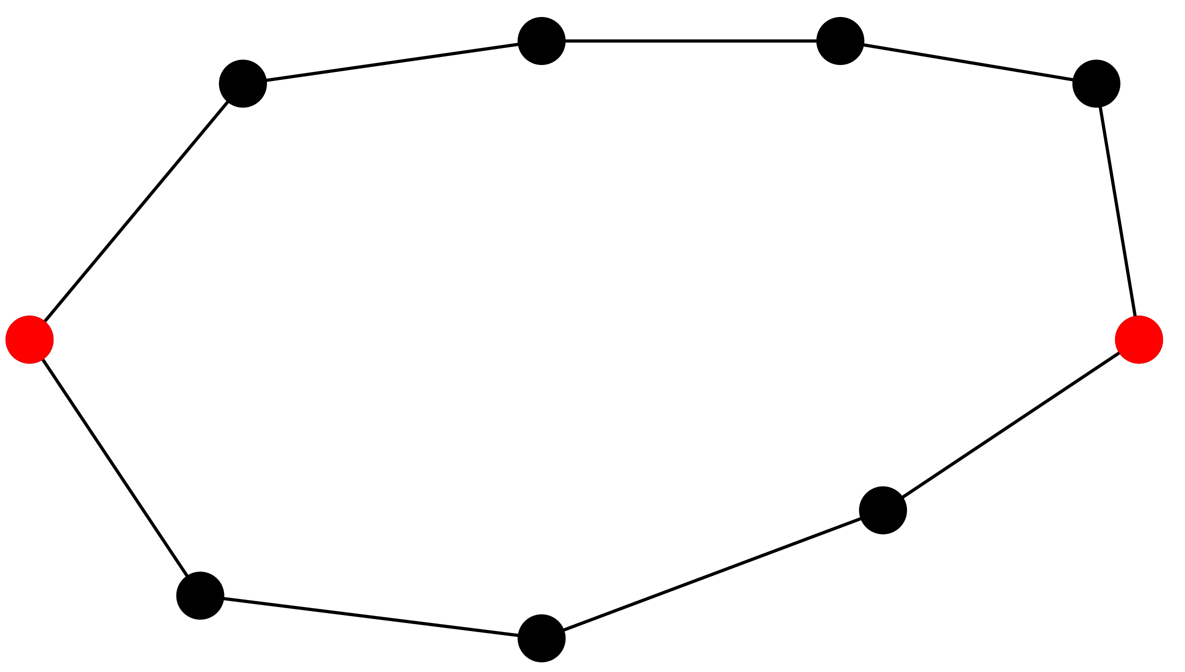
Algorithm of Biniaz et al. (2017):



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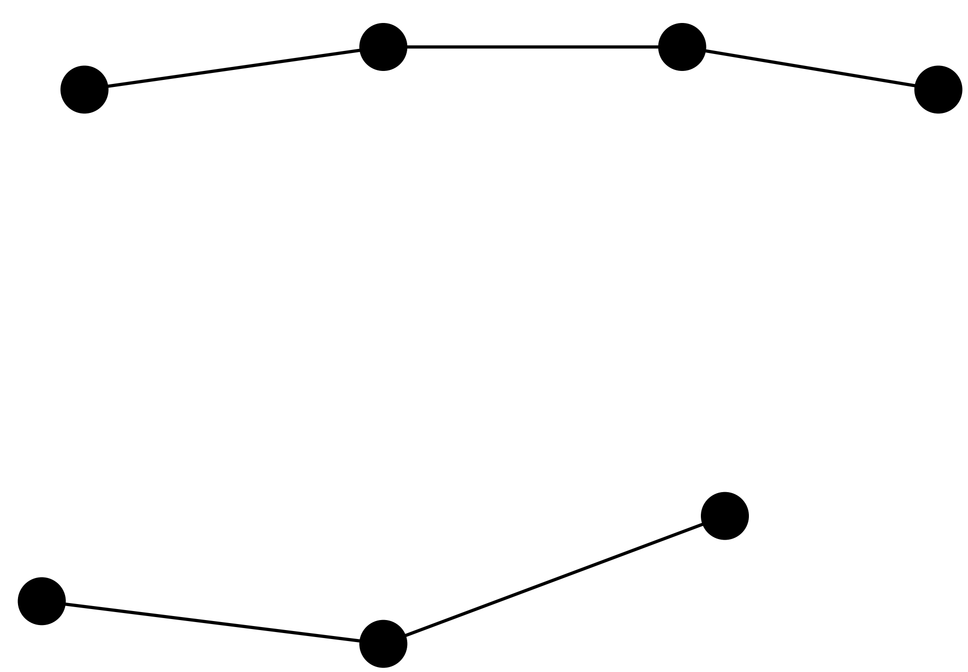

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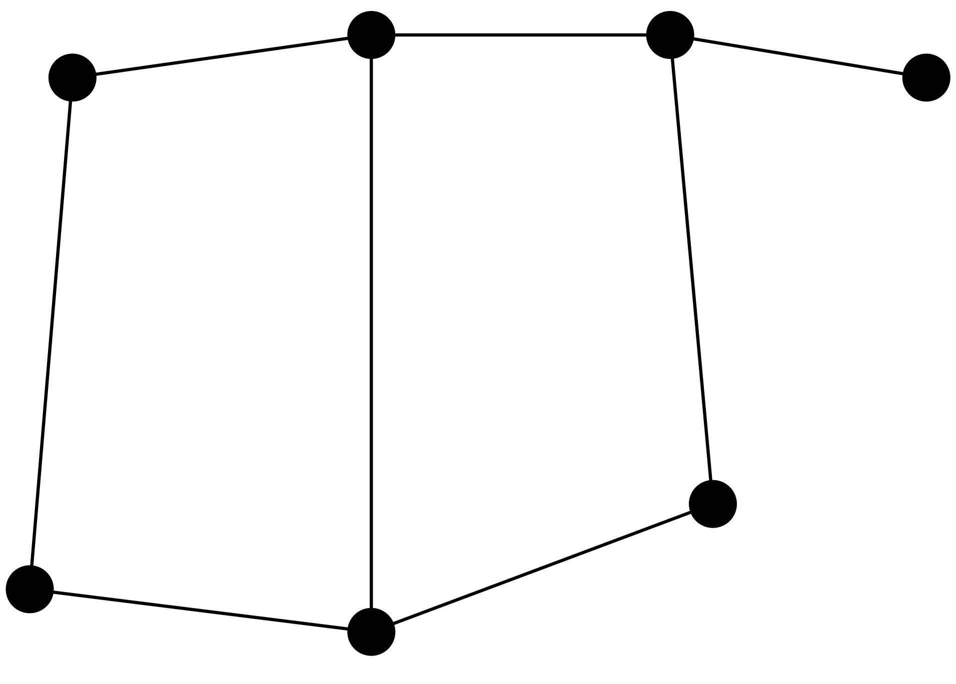

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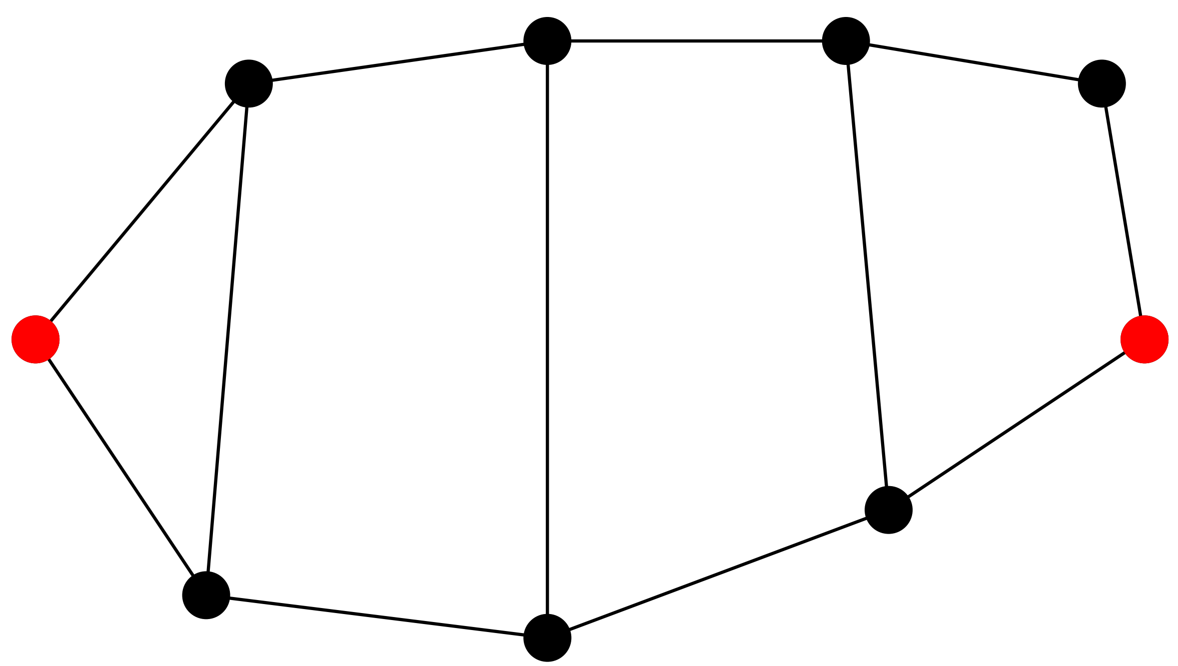

Open Question:

What is the best bound for bounded degree spanners?

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- Kanj et al. showed that degree 3 is optimal when planarity is a constraint by showing that:
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- Dumitrescu and Ghosh (2018) showed a lower bound of 2.732 for max degree 3 planar graphs.

Open Question:

What is the best bound for bounded degree spanners?

For plane max deg 3, the bound lies between 2.73 and 5.19.

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Thank you.